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A Belief Similarity Measure for Dempster-Shafer Evidence Theory and Application in Decision Making

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ABSTRACT

How to effectively deal with uncertain and imprecise information in decision making is a complex task. Dempster-Shafer evidence theory (DSET) is widely used for handling such challenges due to its ability to model uncertainty and imprecision. However, Dempster's rule can sometimes yield counterintuitive results when dealing with highly conflicting evidence. In this paper, we introduce a novel belief sine similarity measure, called BS^2M, which effectively measures the discrepancy between different pieces of evidence. We also establish that BS^2M possesses important properties such as boundedness, symmetry, and non-degeneracy. Building upon BS^2M, we present a new method for decision making. The proposed method considers both the credibility and the information volume of each evidence, providing a more comprehensive reflection of their importance. To validate our method, we conduct experiment in target recognition application, demonstrating the effectiveness and rationality of the proposed method.

1. Introduction

How to deal with uncertain and imprecise information in decision making has emerged as a prominent and important topic, which has gained much attention [1-3]. At present, to solve this problem, a plenty of theories have developed, including fuzzy set theory [4-6], neutrosophic set theory [7,8], intuitionistic fuzzy set [9,10], fermatean fuzzy set [11,12]], N-soft [13,14], Dempster-Shafer evidence theory [15-17], rough set theory [18,19].

Among them, Dempster-Shafer evidence theory (DSET) [20], as a powerful tool for modeling uncertain and imprecise information, which has been successfully applied in pattern recognition [21-23], fault diagnosis [24,25], information fusion [26,27] and image analysis [28,29]. DSET provides

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a richer representation of uncertainty by allowing belief values to be assigned to sets of propositions, rather than just individual events [15]. This granularity has made DSET particularly attractive for complex decision-making tasks, offering a more comprehensive understanding of uncertainty and imprecision. Additionally, DST allows for the combination of the evidences using Dempster's rule, which abides by the associative and commutative laws and is especially useful for multisource data fusion [30].

In certain situations, however, Dempster's rule may produce counterintuitive or unacceptable results, especially when combining highly conflicting evidence [31]. To avoid this flaw, some strategies to avoid or mitigate the counterintuitive behaviors have been presented, mainly divide into two types: One is to modify Dempster's rule, including the Yager's rule [32], Dubois and Prade's rule [33] and Smets's rule [34]. These modified rules provide alternative ways to combine evidence and can sometimes yield more intuitive results than Dempster's rule. Nevertheless, they often break the associative and commutative laws of Dempster's rule and are therefore sometimes limited in application scenarios.

The other is to modify the evidence sources, a lot of studies tend to pre-process the evidence before using Dempster's rule [35-38]. For instance, Murphy's average fusion rule [37], advocates for the computation of the average belief values across various evidences to formulate a new evidence. However, this method often overlooks the varying significance of distinct evidences in practical applications, rendering the equal treatment of each evidence in the fusion process somewhat unjustified. To address this shortcoming, Deng et al. [39] enhanced Murphy's method by integrating the Jousselme distance to quantify the similarity amongst diverse evidences. Building upon this, Jiang et al. [40] introduced an evidential correlation coefficient, aiming to account for the conflicts arising between evidences. Xiao [41] proposed a belief Jenson-Shannon divergence to measure the discrepancy between the evidences and employed belief entropy to calculate the uncertainty of the evidence itself. Recently, Kaur and Srivastava [42] introduced a new divergence to consider the discrepancy between the evidences. Regrettably, these methods overlook the internal variations within propositions, indiscriminately equating multiple propositions with their singleton counterparts. Therefore, how to effectively measure the discrepancy between the pieces of evidence is still a challenging issue.

In this paper, we present a belief similarity measure based on the sine function, called BS^2M, to manage the discrepancy between the evidences in DSET. The BS^2M takes into account the number of possible propositions, which makes them more suitable for the similarity measure between evidences. Moreover, we display that BS^2M has some interesting properties. Finally, we devise a decision making method under DSET. The main contributions are concluded as follows:

- Two new BS^2M is introduced based on the sine function to consider the similarities between the evidences.
- The proposed BS^2M shows several advantageous properties, such as boundedness, symmetry, and non-degeneracy, which make them attractive and powerful solutions for evaluating discrepancies.
- A novel decision making method is developed, utilizing the proposed measure and belief entropy.
- The effectiveness of the proposed decision making method is validated through its application.

The remaining sections of this paper are organized as follows. Section 2 offers a brief introduction to the DSET. In Section 3, two new belief sine similarity measures are proposed. Section 4 presents a decision making method, which is based on the proposed measures and belief entropy. The effectiveness of the proposed method is tested by one application in Section 5. Finally, Section 6 concludes the paper.

2. Preliminaries

Definition 1 (Framework of discernment) *Suppose Θ be a set of mutually exclusive and exhaustive elements, which is called the framework of discernment (FOD) and denoted by:*

$$\Theta = \{\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_N\} \tag{1}$$

In DSET, the power-set of Θ is depicted as 2^Θ:

$$2^\Theta = \{\emptyset, \{\mathcal{X}_1\}, \{\mathcal{X}_2\}, ..., \{\mathcal{X}_N\}, \{\mathcal{X}_1, \mathcal{X}_2\}, ..., \Theta\} \tag{2}$$

where $\{\mathcal{X}_i\}$ and $\{\mathcal{X}_i, \mathcal{X}_j\}$ are the singleton and multiple propositions, $\emptyset$ is an empty set.

Definition 2 (Basic belief assignment) *In the FOD Θ, a basic belief assignment (BBA)$\mathbf{m}$, also called mass function, is a mapping from 2^Θ to $[0, 1]$, which satisfies:*

$$\begin{cases} \sum\limits_{\mathcal{X}_i \subseteq \Theta} m(\mathcal{X}_i) = 1 \\ m(\emptyset) = 0 \end{cases} \tag{3}$$

where $m(\mathcal{X}_i)$ denotes the belief value to $\{\mathcal{X}_i\}$.

Definition 3 (Dempster's rule) *Let $\mathbf{m}_1$ and $\mathbf{m}_2$ be two independent BBAs on Θ, Dempster's rule is described as:*

$$m(\mathcal{X}_i) = \begin{cases} 0, & \mathcal{X}_i = \emptyset \\ \frac{\sum\limits_{\mathcal{X}_j \cap \mathcal{X}_k = \mathcal{X}_i} m_1(\mathcal{X}_j) m_2(\mathcal{X}_k)}{1-K}, & \mathcal{X}_i \neq \emptyset \end{cases} \tag{4}$$

with

$$K = \sum_{\mathcal{X}_j \cap \mathcal{X}_k = \emptyset} m_1(\mathcal{X}_j) m_2(\mathcal{X}_k) \tag{5}$$

where K denotes the conflict coefficient between $\mathbf{m}_1$ and $\mathbf{m}_2$.

Definition 4 (Deng entropy) *Deng [43] proposed the concept of Deng entropy, which is defined as follows:*

$$\boldsymbol{E}_d(\mathbf{m}) = -\sum_{\mathcal{X}_i \subseteq \Theta} m(\mathcal{X}_i) \log \frac{m(\mathcal{X}_i)}{2^{|\mathcal{X}_i|} - 1} \tag{6}$$

3. Proposed Belief Similarity Measure

In DSET, how to effectively measure similarities between the evidences remains an open issue. In this section, a new belief similarity measures is suggested to handle the above question. Moreover, several properties of the proposed belief similarity measure are explored.

Definition 5 *(***Belief sine similarity measure***) Let $\mathbf{m}_1$ and $\mathbf{m}_2$ are two BBAs on Ω, the belief sine similarity measure (BS^2M) between $\mathbf{m}_1$ and $\mathbf{m}_2$ is defined as:*

$$BS^2M(\mathbf{m}_1, \mathbf{m}_2) = 1 - \sin\left(\frac{\pi}{4} \sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right) \tag{7}$$

where the term $2^{|\mathcal{X}_i|} - 1$ considers all the number of possible propositions, thereby incorporating the scale of the FOD's impact. Compared with the previous works such as [1,41,42], BS^2M can reasonably calculate similarities between two BBAs and avoid the negative impact of ignoring multiple propositions.

Theorem 1 *The proposed BS^2M satisfies the following properties:*

1. *Symmetry:* $BS^2M(\mathbf{m}_1, \mathbf{m}_2)$ = $BS^2M(\mathbf{m}_2, \mathbf{m}_1)$.
2. *Bounded:* $0 \le BS^2M(\mathbf{m}_1, \mathbf{m}_2) \le 1$.
3. *Non-degeneracy:* $BS^2M(\mathbf{m}_1, \mathbf{m}_2) = 1$ *if and only if* $\mathbf{m}_1 = \mathbf{m}_2$.

Proof 1 *For two BBAs $\mathbf{m}_1$ and $\mathbf{m}_2$ on Ω, we have:*

$$BS^2M(\mathbf{m}_1, \mathbf{m}_2) = 1 - \sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right)$$

Clearly, we can get the following:

$$0 \le \sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1} \le 2$$

and

$$0 \le \frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1} \le \frac{\pi}{2}$$

For $\sin(x)$, $x \in [0, \frac{\pi}{2}]$, its range is always positive and within $[0, 1]$. Hence, we have $0 \le BS^2M(\mathbf{m}_1, \mathbf{m}_2) \le 1$.

Proof 2 *For two arbitrary BBAs $\mathbf{m}_1$ and $\mathbf{m}_2$ in Ω, we have:*

$$BS^2M(\mathbf{m}_1, \mathbf{m}_2) = 1 - \sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right)$$

$$BS^2M(\mathbf{m}_2, \mathbf{m}_1) = 1 - \sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_2(\mathcal{X}_i) - m_1(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right)$$

We can easily obtain $BS^2M(\mathbf{m}_1, \mathbf{m}_2) = BS^2M(\mathbf{m}_2, \mathbf{m}_1)$.

Proof 3 *For two same BBAs $\mathbf{m}_1$ and $\mathbf{m}_2$ in Ω, we have:*

$$\begin{aligned} BS^2M(\mathbf{m}_1, \mathbf{m}_2) &= 1 - \sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right) \\ &= 1 - \sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_1(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right) \\ &= 1 \end{aligned}$$

Contrariwise, assume that $BS^2M(\mathbf{m}_1, \mathbf{m}_2) = 1$, we thus have:

$$1 - \sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right) = 1$$

and

$$\sin\left(\frac{\pi}{4}\sum_{\mathcal{X}_i \subseteq \Omega} \frac{|m_1(\mathcal{X}_i) - m_2(\mathcal{X}_i)|}{2^{|\mathcal{X}_i|} - 1}\right) = 0$$

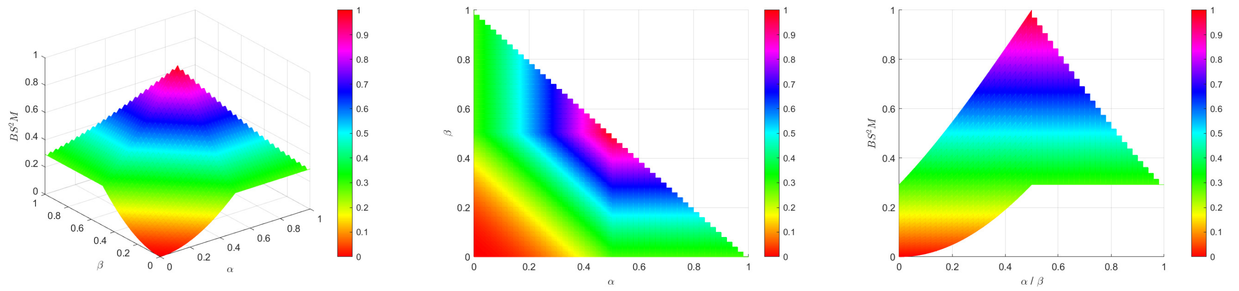
We can easily conclude $\mathbf{m}_1 = \mathbf{m}_2$. Hence, we can prove the property of non-degeneracy.

Here, several examples are used to illustrate the properties of BS^2M.

Example 1 *Let m_1 and m_2 be two BBAs in $\Theta = \{\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3\}$.*

$$\begin{aligned} m_1 &: \ m_1(\{\mathcal{X}_1\}) = \alpha, \quad m_1(\{\mathcal{X}_2\}) = \beta, \quad m_1(\{\mathcal{X}_3\}) = 1 - \alpha - \beta \\ m_2 &: \ m_2(\{\mathcal{X}_1\}) = 0.5, \quad m_2(\{\mathcal{X}_2\}) = 0.5 \end{aligned}$$

where $0 \leq \alpha, \beta \leq 1$, and $0 \leq \alpha + \beta \leq 1$.



(a) The BS^2M varying with α and β. (b) Variation of α and β. (c) The BS^2M varying with α/β.

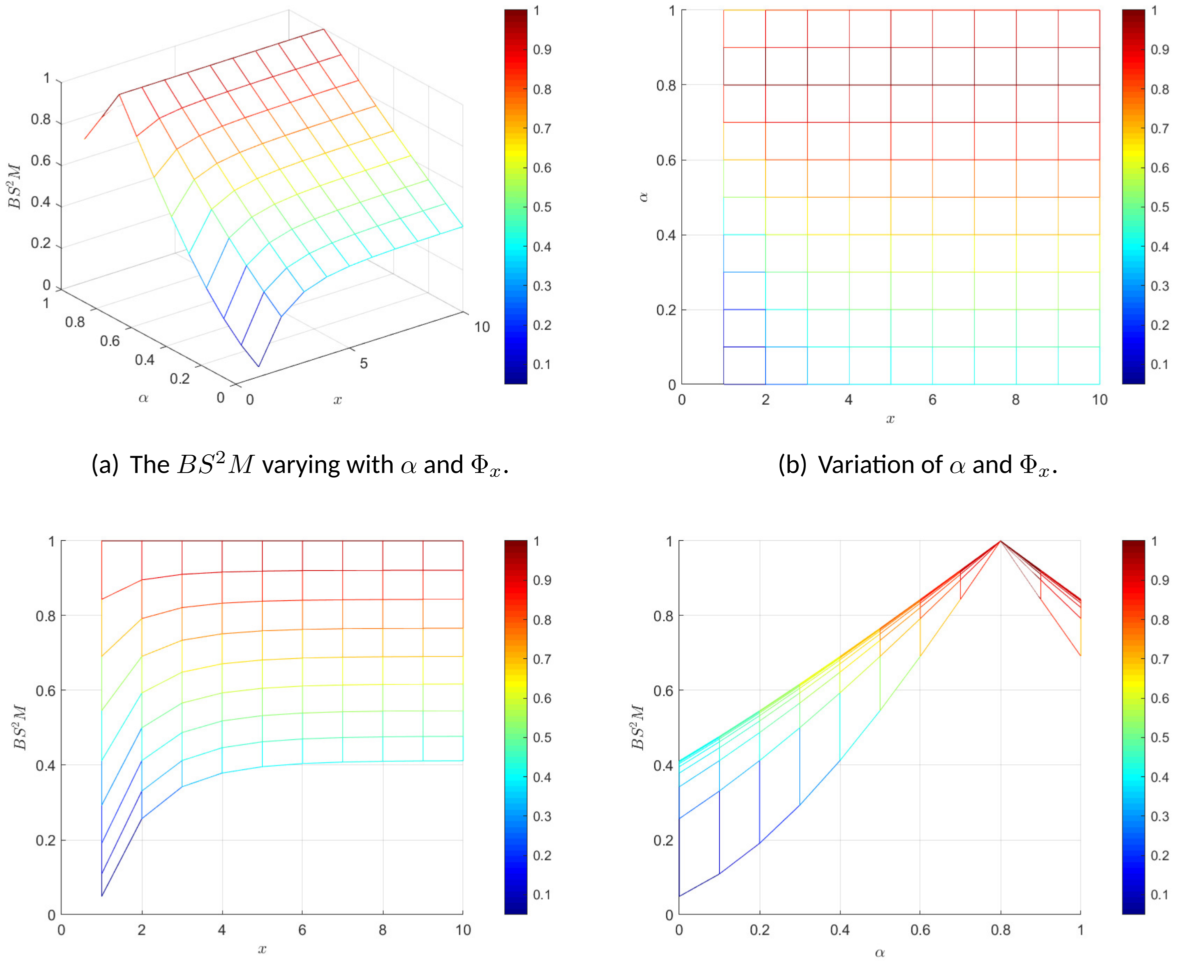
Figure 1: The results of BS^2M varying with α and β in *Example 1*.

As shown in Figure 1, when $\alpha = 0.5$ and $\beta = 0.5$, we have $m_1(\{\mathcal{X}_1\}) = 0.5$, $m_1(\{\mathcal{X}_2\}) = 0.5$, so $\mathbf{m}_1 = \mathbf{m}_2$, BS^2M has the maximum belief values of 1. When $\alpha = 0$ and $\beta = 0$, we have $m_1(\{\mathcal{X}_1\}) = 0$, $m_1(\{\mathcal{X}_2\}) = 0$, $m_1(\{\mathcal{X}_3\}) = 1$, in which case $\mathbf{m}_1$ and $\mathbf{m}_2$ are in complete conflict, BS^2M gets the minimum belief values of 0. Furthermore, BS^2M always ranges between $[0, 1]$ regardless of how α and β change. Besides, we can also observe that $BS^2M(\mathbf{m}_1, \mathbf{m}_2)$ = $BS^2M(\mathbf{m}_2, \mathbf{m}_1)$. Hence, this example shows the properties of symmetry, bounded and non-degenracy.

Example 2 *Suppose that $\mathbf{m}_1$ and $\mathbf{m}_2$ are two BBAs in $\Theta = \{\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_{10}\}$.*

$$\begin{aligned} \mathbf{m}_1 &: \ m_1(\{\mathcal{X}_2\}) = \alpha, \quad m_1(\Phi_x) = 1 - \alpha \\ \mathbf{m}_2 &: \ m_2(\{\mathcal{X}_2\}) = 0.8, \quad m_2(\Phi_x) = 0.2 \end{aligned}$$

where $0 \leq \alpha \leq 1$, and Φ_x denotes the set of element, range from $\{\mathcal{X}_1\}$ to $\{\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_{10}\}$.



(a) The BS^2M varying with α and Φ_x.

(b) Variation of α and Φ_x.

(c) The BS^2M varying with α.

(d) The BS^2M varying with Φ_x.

Figure 2: The results of BS^2M varying with α and Φ_x in *Example 2*.

As shown in Figure 2, when $\alpha = 0.8$, we obtain $\mathbf{m}_1 = \mathbf{m}_2$, and BS^2M obtains the maximum belief values of 1. Besides, BS^2M always ranges between $[0, 1]$ regardless of how α and Φ_x change. Example 1 *and* Example 2 *demonstrate that BS^2M can effectively measure the similarity between different propositions of BBAs.*

4. Proposed decision making method

In this section, we introduce an advanced decision making method, leveraging the belief sine similarity measure and belief entropy, tailored to optimally merge highly conflicting evidence. Our method uniquely incorporates both credibility and the volume of information, ensuring a more nuanced evaluation of each evidence's significance. Initially, we employ BS^2m to determine the credibility weight for each evidence. Subsequently, belief entropy is harnessed to ascertain the weight linked to the information volume of each evidence. Conclusively, comprehensive weights guide the creation of weighted average evidence, with the final fusion outcome derived using Dempster's rule.

Let us consider p independent evidences $\mathbf{m}_k(k = 1, \cdots, p)$ on $\Theta = \{\mathcal{X}_1, \cdots, \mathcal{X}_N\}$.

Step 1: Calculate the similarity between $\mathbf{m}_k(k = 1, \cdots, p)$ and $\mathbf{m}_l(l = 1, \cdots, p)$, denoted as $BS^2M(\mathbf{m}_k, \mathbf{m}_l)$, by using Eq. (7). The similarity matrix $\boldsymbol{SM}_{p\times p}$ is constructed as:

$$\boldsymbol{SM}_{p\times p} = \begin{bmatrix} 1 & BS^2M(\mathbf{m}_1, \mathbf{m}_2) & \dots & BS^2M(\mathbf{m}_1, \mathbf{m}_p) \\ BS^2M(\mathbf{m}_2, \mathbf{m}_1) & 1 & \dots & BS^2M(\mathbf{m}_2, \mathbf{m}_p) \\ \vdots & \ddots & \vdots & \\ BS^2M(\mathbf{m}_p, \mathbf{m}_1) & BS^2M(\mathbf{m}_p, \mathbf{m}_2) & \dots & 1 \end{bmatrix} \tag{8}$$

Step 2: Calculate the support degree $\boldsymbol{S}(\mathbf{m}_k)$ of $\mathbf{m}_k$ as:

$$\boldsymbol{S}(\mathbf{m}_k) = \sum_{l=1, l\neq k}^{p} BS^2M(\mathbf{m}_k, \mathbf{m}_l) \tag{9}$$

Step 3: Obtain the credibility weight $\boldsymbol{W}_C(\mathbf{m}_k)$ of $\mathbf{m}_k$ as:

$$\boldsymbol{W}_C(\mathbf{m}_k) = \frac{\boldsymbol{S}(\mathbf{m}_k)}{\sum_{k=1}^{p} \boldsymbol{S}(\mathbf{m}_k)} \tag{10}$$

Step 4: Calculate the belief entropy $\boldsymbol{E}_d(\mathbf{m}_k)$ of $\mathbf{m}_k$ as:

$$\boldsymbol{E}_d(\mathbf{m}_k) = -\sum_{\mathcal{X}_i \subseteq \Theta} m_k(\mathcal{X}_i) \log \frac{m_k(\mathcal{X}_i)}{2^{|\mathcal{X}_i|} - 1} \tag{11}$$

Step 5: Compute the information volume $\boldsymbol{IV}(\mathbf{m}_k)$ of $\mathbf{m}_k$ as:

$$\boldsymbol{IV}(\mathbf{m}_k) = \exp\left(\boldsymbol{E}_d(\mathbf{m}_k)\right), \forall k = 1, \dots, p \tag{12}$$

Step 6: Obtain the information volume weight $\boldsymbol{W}_{IV}(\mathbf{m}_k)$ of $\mathbf{m}_k$ as:

$$\boldsymbol{W}_{IV}(\mathbf{m}_k) = \frac{\boldsymbol{IV}(\mathbf{m}_k)}{\sum_{k=1}^{p} \boldsymbol{IV}(\mathbf{m}_k)} \tag{13}$$

Step 7: Obtain the comprehensive weight $\boldsymbol{W}(\mathbf{m}_k)$ of $\mathbf{m}_k$ as:

$$\boldsymbol{W}(\mathbf{m}_k) = \frac{\boldsymbol{W}_C(\mathbf{m}_k) \times \boldsymbol{W}_{IV}(\mathbf{m}_k)}{\sum_{k=1}^{n} \boldsymbol{W}_C(\mathbf{m}_k) \times \boldsymbol{W}_{IV}(\mathbf{m}_k)} \tag{14}$$

Step 8: Generate the weighted average evidence $\bar{\mathbf{m}}$ as:

$$\bar{m}(\mathcal{X}_i) = \sum_{k=1}^{n} \boldsymbol{W}(\mathbf{m}_k) \times m_k(\mathcal{X}_i) \tag{15}$$

Step 9: Utilize Eq. (4) to fuse $\bar{\mathbf{m}}$ $n - 1$ times.

5. Application

To validate the effectiveness of the proposed method against other competitive techniques, we employed a target recognition application focusing on aircraft types, as delineated in [44]. In this scenario, five radar sensors ($\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{S}_3$, $\mathbb{S}_4$ and $\mathbb{S}_5$) gather data, which is subsequently represented as basic belief assignments (BBAs). The potential aircraft types under consideration are the airliner airliner $\{\mathcal{X}_1\}$, bomber $\{\mathcal{X}_2\}$ and fighter $\{\mathcal{X}_3\}$, constituting the framework of discernment $\Theta = \{\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3\}$. The BBAs derived from each sensor's data are articulated in Table 1. Notably, all BBAs align with target $\{\mathcal{X}_1\}$ with the exception of $\mathbf{m}_2$. Given its stark deviation from the consensus, $\mathbf{m}_2$ is deemed an unreliable evidence due to its heightened conflict with the remaining evidences.

Table 1: BBAs modeled from sensors in target recognition

BBAs	$\{\mathcal{X}_1\}$	$\{\mathcal{X}_2\}$	$\{\mathcal{X}_3\}$	Θ
$\mathbb{S}_1$: $\mathbf{m}_1(\cdot)$	0.40	0.60	0.00	0.00
$\mathbb{S}_2$: $\mathbf{m}_2(\cdot)$	0.00	0.70	0.30	0.00
$\mathbb{S}_3$: $\mathbf{m}_3(\cdot)$	0.85	0.00	0.00	0.15
$\mathbb{S}_4$: $\mathbf{m}_4(\cdot)$	0.40	0.60	0.00	0.00
$\mathbb{S}_5$: $\mathbf{m}_5(\cdot)$	0.75	0.00	0.00	0.25

The results of different methods are detailed in Table 2. Notably, Dempster's rule exclusively favors target $\{\mathcal{X}_2\}$, highlighting its inherent difficulty in managing evidence with significant conflict. In contrast, the proposed method successfully discerns target $\{\mathcal{X}_1\}$, aligning seamlessly with the findings from various alternative methods. Additionally, Table 2 displays how the results shift with an increasing amount of evidence. Dempster's rule persistently endorses $\{\mathcal{X}_2\}$ incorrectly, whereas the belief value for $\{\mathcal{X}_1\}$ progressively ascends when applying other methods. Among them, the proposed method yields the most substantial belief value, peaking at 0.8866 for $\{\mathcal{X}_1\}$, underscoring its practical applicability and efficacy.

6. Conclusion

This paper proposes a new belief similarity measure for capturing conflicts between evidences based on DSET. The proposed measure can reasonably distinguish the effects of singleton and multiple propositions, and satisfy desirable properties. Moreover, a decision making method is developed based on the proposed measure and belief entropy, offering an effective approach for resolving conflicts and facilitating decision-making processes. Numerical examples and application results verify the efficiency and potential of the proposed method in decision-making. In future studies, we plan to extend the application of the proposed method to other domains that involve uncertainty and imprecision, such as risk analysis and medical diagnosis.

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Conflicts of Interest

The author declares no conflicts of interest.

Table 2: Fusion results of different methods in pattern classification

Methods	Focal element	$\mathbf{m}_{1,2}$	$\mathbf{m}_{1,2,3}$	$\mathbf{m}_{1,2,3,4}$	$\mathbf{m}_{1,2,3,4,5}$
Dempster's rule [20]	$\{\mathcal{X}_1\}$	0.0000	0.0000	0.0000	0.0000
	$\{\mathcal{X}_2\}$	**1.0000**	**1.0000**	**1.0000**	**1.0000**
	$\{\mathcal{X}_3\}$	0.0000	0.0000	0.0000	0.0000
	Θ	0.0000	0.0000	0.0000	0.0000
Murphy's method [37]	$\{\mathcal{X}_1\}$	0.0825	0.4663	0.6263	**0.7273**
	$\{\mathcal{X}_2\}$	**0.8711**	**0.5182**	**0.6263**	0.2720
	$\{\mathcal{X}_3\}$	0.0464	0.0149	0.0014	0.0007
	Θ	0.0000	0.0000	0.0000	0.0000
Deng et al.'s method [39]	$\{\mathcal{X}_1\}$	0.0825	0.3501	0.2440	**0.7261**
	$\{\mathcal{X}_2\}$	**0.8711**	**0.6389**	**0.7554**	0.2736
	$\{\mathcal{X}_3\}$	0.0464	0.0108	0.0006	0.0003
	Θ	0.0000	0.0000	0.0000	0.0000
Lin et al.'s method [1]	$\{\mathcal{X}_1\}$	0.0825	0.3991	0.2805	**0.6901**
	$\{\mathcal{X}_2\}$	**0.8711**	**0.5905**	**0.7190**	0.3096
	$\{\mathcal{X}_3\}$	0.0464	0.0101	0.0006	0.0003
	Θ	0.0000	0.0000	0.0000	0.0000
Jiang's method [40]	$\{\mathcal{X}_1\}$	0.0825	0.3405	0.2499	**0.7328**
	$\{\mathcal{X}_2\}$	**0.8711**	**0.6512**	**0.7496**	0.2669
	$\{\mathcal{X}_3\}$	0.0464	0.0081	0.0005	0.0002
	Θ	0.0000	0.0002	0.0000	0.0000
Xiao's method [41]	$\{\mathcal{X}_1\}$	0.0825	0.4404	0.2887	**0.8393**
	$\{\mathcal{X}_2\}$	**0.8673**	**0.5526**	**0.7111**	0.1605
	$\{\mathcal{X}_3\}$	0.0424	0.0067	0.0002	0.0002
	Θ	0.0000	0.0003	0.0000	0.0001
Gao and Xiao's method [36]	$\{\mathcal{X}_1\}$	0.0903	0.4509	0.2991	**0.8504**
	$\{\mathcal{X}_2\}$	**0.8673**	**0.5421**	**0.7007**	0.1494
	$\{\mathcal{X}_3\}$	0.0424	0.0066	0.0002	0.0002
	Θ	0.0000	0.0004	0.0000	0.0001
Proposed method	$\{\mathcal{X}_1\}$	0.0903	0.3072	0.2110	**0.8866**
	$\{\mathcal{X}_2\}$	**0.8673**	**0.6855**	**0.7889**	0.1131
	$\{\mathcal{X}_3\}$	0.0424	0.0072	0.0002	0.0002
	Θ	0.0000	0.0001	0.0000	0.0001

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