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Improved q-Rung Orthopair Fuzzy WASPAS Method Based on Softmax Function and Frank Operations for Investment Decision of Community Group-Buying Platform

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ABSTRACT

The traditional WASPAS (Weighted Aggregated Sum Product ASsessment) method has attracted widely attention. Unfortunately, the decision-makers' decision attitude or risk preference is ignored in the existing WASPAS methods. To overcome this shortcoming, this paper embedded expert's decision attitude with risk preference into WASPAS method for solving multi-attribute group decision-making problems with q-rung orthopair fuzzy (q-ROF) information. We develop the q-ROF Frank softmax weighted averaging (q-ROFFSWA) and q-ROF Frank softmax weighted geometric (q-ROFFSWG) operators based on Frank operations and softmax function. The relevant properties and particular cases are explored, and the monotonicity of these operators' score functions is analyzed. Then, the q-ROF multi-attribute group decision-making framework based on the improved WASPAS method is constructed. The weighted sum model and weighted product model in traditional WASPAS are replaced by the two proposed aggregation operators. The q-ROF distance measure is utilized to de fuzzy the performance values of alternatives. And a compromise function between optimistic and pessimistic decision attitudes with risk preferences is proposed. Lastly, the presented method is implemented in a real case of investment decision of community group-buying (CGB) platform, and sensitivity analysis and comparative study with existing methods are conducted to verify the practicality, robustness and effectiveness.

1 Introduction

The multi-attribute group decision-making (MAGDM) is a cross research direction of group decision-making and multi-attribute decision-making (MADM), which integrates the alternative preference information given by multiple DMs into group preference information, and the constructed theory is used to selection the best of limited options [1]. Recently, the MAGDM has turn into a hot topic in modern decision-making field. However, the actual group decision-making problems have become more complicated with the rapid development of economy and society.

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Many scholars face great challenges in depicting the ambiguity, uncertainty and personality preference of individual opinions and views in the evaluation process. Fuzzy set (FS) [2] is an effective means for evaluation information description and widely used to deal with information modeling problems, but FS is not competent to describe the uncertainty of human cognition of things, because there is only MD μ (x) (0≤ μ (x)≤1) in FS. Atanassov [3] proposed a binary form composed of MD μ (x) and ND ν (x) (0≤ ν (x) ≤1), and called it intuitionistic fuzzy set (IFS). Compared with FS, IFS can describe more detailed assessment information. The IFS is characterized by μ (x) + ν (x) ≤ 1. However, IFS cannot be used to solve the decision problems where μ (x) + ν (x) > 1. For this purpose, Yager [4] extended the Pythagorean fuzzy set (PyFS), whose μ (x) and ν (x) meet the conditions: μ 2(x) + ν 2(x) ≤ 1. Yager [5] further designed generalized q-ROFSs based on the PyFSs to meet μ q(x) + ν q(x) ≤ 1 (q≥1). Obviously, the q-ROFSs have more decision space and more free evaluation information than IFSs and PyFSs, so that experts can express their individual views and preferences more accurately. Therefore, the generalized q-ROFSs can depict the nature of vagueness and uncertainty of evaluation information more effectively.

The WASPAS method was first developed in 2012 [6], which integrates two basic MADM models based on the utility theory, namely weighted sum model (WSM) and weighted product model (WPM). This methodology is an uncomplicated decision method that combines WSM and WPM through linear combination system and adopts the concept of ranking accuracy. These results are more accurate and stable than those obtained by traditional WSM or WPM [7]. Based on the above advantages, many scholars have paid attention to WASPAS method in various fuzzy environments, e.g., nuetrosophic sets, hesitant fuzzy sets (HFSs) , probabilistic linguistic term sets (PLTSs) , rough sets (RSs) , spherical fuzzy sets (SFSs) , picture fuzzy sets (PFSs) , PyFSs and q-ROFSs [8-20], etc. Some existing WASPAS methods are listed in Table 1. However, there are still some shortcomings in the existing methods: (1) The WSM and WPM adopt basic Algebraic operational laws (AOLs), and do not consider the priority relationship between variables when aggregating information, so they cannot reflect the actual situation of decision. (2) The WSM and WPM in the existing WASPAS methods do not contain any parameters, which makes the traditional WASPAS method cannot express the decision preference and intention of DMs, and also cannot present the decision flexibility. Although Pamucar *et al.*[21] incorporated HM operator into WASPAS method in fuzzy environment and can make its flexible decision-making though parameters, the intention and viewpoints of DMs cannot be expressed. (3) Rani *et al.* [20] and Ayyildiz *et al.* [19] used score function [22] to de fuzzy the results of WSM and WPM under the Pythagorean fuzzy environment, but the score function does not consider the influence of abstention or hesitancy degree in PyFNs, which means that part of the loss of information may result in the two PyFNs cannot be accurately distinguish [23,24]. This situation also exists in the q-ROF environment. Therefore, it is necessary to improve the traditional WASPAS method in this paper to make up for the above shortcomin.

Table 1
Existing WASPAS methods in different decision environment

Year	References	Information type	Decision type	Fuse tool for individual evaluation information	WSM/WPM OLs	Flex.	De fuzzification technique	Application area
2015	[8]	SVNSs	Single	-	AOLs	NO	Score function	Waste incineration sit evaluation
2016	[25]	IT2FSs	Group	Arithmetic mean	AOLs	NO	Score function	Green supplier selection
2017	[9]	IVNSs	Single	-	AOLs	NO	Score function	Solar-wind power station location
2018	[14]	RSs	Group	RDWGA	AOLs	NO	-	3PL provider selection
2018	[26]	IVIFSs	Group	IVIFWA	AOLs	NO	Score function	Reservoir flood control
2019	[10]	DHHFLTSs	Group	DHHFHA	AOLs	NO	Transform to crisp value	Risk management technique
2019	[12]	PLTSs	Group	PLSWG	AOLs	NO	Transform to crisp value	Numerical example
2019	[11]	HFSs	Group	HFWA	AOLs	NO	-	Green supplier selection
2019	[15]	IVRNs	Group	Arithmetic mean	AOLs	NO	Transform to crisp value	third-party logistics provider selection
2019	[27]	LNNs	Group	LNNNWGBM	AOLs	NO	Score function	Transport of hazardous goods
2020	[16]	SFSs	Group	SWAM	AOLs	NO	Score function	Numerical example
2020	[17]	SFSs	Single	-	AOLs	NO	Score function	Petrol station location
2020	[20]	IFT2Ss(PyFSs)	Group	PyFWA	AOLs	NO	Score function	Physician selection
2020	[21]	FSs	Group	HM	AOLs	YES	-	Airport ground access mode selection
2020	[28]	Z-numbers	Group	Arithmetic mean	AOLs	NO	Arithmetic mean	HSE risk evaluation
2020	[29]	q-ROFSs	Group	q-ROFWA	AOLs	NO	Score function	Fuel technology selection
2021	[13]	UPLTSs	Group	UPLWA or UPLWG	AOLs	NO	Score function	Numerical example
2021	[19]	PyFSs	Group	Weighted sum method	AOLs	NO	Score function	Refugee camp location
2022	[18]	PFS	Single	-	AOLs	NO	Score function	Numerical example
2023	[30]	PDHLTSs	Group	Arithmetic me an	AOLs	NO	Score function	Risk assessment
-	This article	q-ROFSs	Group	q-ROFFSWA and q-ROFFSWG	FOLs	Yes	Distance measure	CGB platform investment decision

Abbr.: SVNSs: single-valued neutrosophic sets; IT2FSs: interval type-2 fuzzy sets; IVNSs: interval-valued neutrosophic sets; IVIFSs: interval-valued intuitionistic fuzzy sets; PLTSs: DHHFLTSs: double hierarchy hesitant fuzzy linguistic term sets; IVRNs: interval-valued rough numbers; LNNs: linguistic neutrosophic numbers; IFT2Ss: intuitionistic fuzzy type-2 sets; UPLTSs: uncertain probabilistic linguistic term sets; RDWGA: rough Dombi weighted geometric aggregator; IVIFWA: interval-valued intuitionistic fuzzy weighted average; PLSWG: probabilistic linguistic simple weighted geometry; DHHFHA:

double hierarchy hesitant fuzzy hybrid aggregation; HFWA: hesitant fuzzy weighted average; LNNNWGBM: linguistic neutrosophic number normalized weighted geometric Bonferroni mean; SWAM: spherical weighted Arithmetic mean; PyFWA: Pythagorean fuzzy weighted average; HM: Heronian mean; q-ROFWA: q-rung orthopair fuzzy weighted average; UPLWA: uncertain probabilistic linguistic weighted average; UPLWG: uncertain probabilistic weighted geometric; FOLs: Frank operation laws; PDHLTS: probabilistic double hierarchy linguistic term sets.

The operational rules and the priority relationship of variables are crucial for assessment information fusion. In terms of operational rules, many scholars have conducted many studies on generalized operation rules in the q-ROFS environment, such as Archimedean, Einstein, Frank, Hamacher and Dombi [31-35]. It is worth noting that Frank T-norm and S-norm [36] is the only operational form with compatibility characteristics in above operations, which has better generality, flexibility and robustness in dealing with information aggregation to overcome the defects of AOLs. Since the Frank T-norm and S-norm can be degenerated into Lukasiewicz and Algebraic operations under special conditions, it has been applied to define the operational laws in various fuzzy theories, such as IFSs, HFSs [37,38], etc. However, there are barely studies on the Frank operations with q-ROFNs [33]. In terms of the priority level, the softmax function is the extension of Logistic regression model on multiple classification problems, which has been widely used on deep learning, decision-making [39-41] and other fields. The softmax function can effectively depict the priority relationship between decision variables in different decision-making environments [40], so it can be applied in the study of various aggregation operators (AOs). For example, Torres *et al.* [41] first extended softmax function to hesitant fuzzy sets and developed some AOs. Yu [40] developed two AOs based on the softamx function in IFSs. Compared with the existing prioritized AOs with q-ROF information [42-43], since the softmax function contains exponential function and a modulation parameter, it not only has the characteristics of non-linearity, monotonicity and boundedness [40], but also can show stronger generalization and decision-making flexibility. However, there is no relevant study on softmax function in q-ROF environment. Therefore, some new AOs need to be developed based on the advantages of FOLs and softmax function.

Based on the above research motivations, some contributions of this article are shown as below:

(1) We propose two AOs based on the FOLs and softmax function, including the q-ROFFSWA and q-ROFFSWG operators. The effective properties and some special cases are discussed, and the monotonicity of these two AOs' score functions is analyzed.

(2) A novel q-ROF MAGDM framework is built based on the improved WASPAS method. In this improved method, the WSM and WPM are substituted by the q-ROFFSWA and q-ROFFSWG operators respectively, the q-ROF distance measure is utilized to de fuzzy the performance values of alternatives, and a new compromise function is constructed between optimistic and pessimistic decision attitude, which contains experts' risk preference.

(3) We apply the proposed methodology to solve the real case of investment decision-making of CGBP considering expert risk preferences, and we validate the effectiveness and feasibility of proposed method through sensitivity analysis and comparison study.

Section 2 briefly introduces the relevant notions, i.e., q-ROFS, softmax function and Frank operations. We proposed the q-ROFFSWA and q-ROFFSWG operators in Section 3. We present a new MAGDM approach based on improved WASPAS method considering the risk preference in Section 4. In Section 5, we study a real case for investment decision of CGB platform to illustrate the presented method, the sensitivity and comparative analysis are conducted. In Section 6, we outline several concluding remarks.

2 Preliminaries

Definition 1. Suppose X={x1, x2,…, xn} is a finite universe [5]. A q-ROFS Δ is defined as

$$\Delta = \left\{ \left\langle x_j, \left(\mu_\Delta \left(x_j \right), \nu_\Delta \left(x_j \right) \right) \right\rangle \middle| x_j \in X \right\} \tag{1}$$

where $\mu_\Delta(x_j)$ and $\nu_\Delta(x_j)$ are the MD and ND of element xj belonging X toΔ, respectively. The abstention degree is $\pi_\Delta\left(x_j\right) = \sqrt[q]{1 - \left(\left(\mu_\Delta \left(x_j \right) \right)^q + \left(\nu_\Delta \left(x_j \right) \right)^q \right)}$. The binary (μ, ν) is called q-ROF number (q-ROFN), it is simply expressed as δ = (μ, ν), where μ,ν∈[0,1] and μq + νq ≤ 1 (q ≥ 1).

Definition 2. Suppose δ = (μ,ν) is a q-ROFN, its score function and accuracy function can be represented as follows [5]:

$$Sc(\delta) = \frac{1 + \mu^q - \nu^q}{2},\ Sc(\delta) \in [0,1] \tag{2}$$

$$Ac(\delta) = \mu^q + \nu^q,\ Ac(\delta) \in [0,1] \tag{3}$$

Definition 3. For two q-ROFNs δ1=(μ1, v1) and δ2=(μ2, v2). Then, (1) if Sc(δ1) > Sc(δ2), then δ1 is larger than δ2; (2) if Sc(δ1) = Sc(δ2), then if Ac(δ1) > Ac(δ2), then δ1 is larger than δ2; and if Ac(δ1) = Ac(δ2), then δ1 and δ2 are equal [44].

Definition 4. The normalized Hamming distance between two q-ROFNs [45], δ1=(μ1, v1) and δ2=(μ2, v2), can be defined by

$$D(\delta_1, \delta_2) = \frac{1}{2}(\left|\mu_1^q - \mu_2^q\right| + \left|\nu_1^q - \nu_2^q\right| + \left|\pi_1^q - \pi_2^q\right|) \tag{4}$$

Definition 5. As a generalized form of Logistic function, the softmax function is defined as [40]:

$$\phi^\kappa(j, T_1, T_2, \ldots, T_n) = \phi_j^\kappa = \frac{\exp\left(T_j / \kappa\right)}{\sum_{j=1}^{n} \exp\left(T_j / \kappa\right)} \quad \kappa > 0 \tag{5}$$

where κ is modulation parameter. For a set of q-ROFNs δj (j=1,2,…,n), the Sc(δj) is the score function of q-ROFN δj, and the Tj is obtained by the following Eq.(6):

$$T_j = \begin{cases} \prod_{l=1}^{j-1} Sc(\delta_j) & j = 2,3,\ldots,n \\ 1 & j = 1 \end{cases} \tag{6}$$

We can find that the value of softmax function is in the range of [0, 1] and satisfies $\sum_{j=1}^{n} \phi_j^\kappa = 1$. Yu [40] and Torres *et al.* [41] both believe that it has the properties of nonlinearity, monotonicity and boundedness.

Definition 6. For any two real numbers x, y∈[0,1], Frank product and Frank sum are described as below [36].

$$T(x,y) = x \otimes_F y = \log_\theta\left(1 + \frac{(\theta^x - 1)(\theta^y - 1)}{\theta - 1}\right); S(x,y) = x \oplus_F y = 1 - \log_\theta\left(1 + \frac{(\theta^{1-x} - 1)(\theta 1 -^y - 1)}{\theta - 1}\right) \tag{7}$$

where θ∈(1,+∞). The Frank operations have two particular cases: (1) If θ→1, the Frank product and Frank sum are reduced to the Algebraic operations, namely T(x, y) = xy and S(x, y) = x+y-xy. (2) If θ→+∞, the Frank product and Frank sum are reduced to the Lukasiewicz operations, namely T(x, y) → max (0, x+y-1) and S(x, y) → min (x+y, 1).

3 The q-ROF Frank Softmax aggregation operators

3.1 The q-ROF FOLs

On the basis of Xing *et al.* [46] and Zhang *et al.* [37], we can extend the FOLs under q-ROF environment.

Definition 7. Suppose δ=(μ, ν), δ1=(μ1, ν1) and δ2=(μ2, ν2) are three q-ROFNs, then the q-ROF FOLs are defined as follows (θ>1, λ>0).

(1) $\delta_1 \oplus_F \delta_2 = \left(\sqrt[q]{1-\log_\theta\left(1+\frac{(\theta^{1-\mu_1^q}-1)(\theta^{1-\mu_2^q}-1)}{\theta-1}\right)}, \sqrt[q]{\log_\theta\left(1+\frac{(\theta^{\nu_1^q}-1)(\theta^{\nu_2^q}-1)}{\theta-1}\right)} \right)$

(2) $\delta_1 \otimes_F \delta_2 = \left(\sqrt[q]{\log_\theta\left(1+\frac{(\theta^{\mu_1^q}-1)(\theta^{\mu_2^q}-1)}{\theta-1}\right)}, \sqrt[q]{1-\log_\theta\left(1+\frac{(\theta^{1-\nu_1^q}-1)(\theta^{1-\nu_2^q}-1)}{\theta-1}\right)} \right)$

(3) $\lambda \cdot_F \delta = \left(\sqrt[q]{1-\log_\theta\left(1+\frac{(\theta^{1-\mu^q}-1)^\lambda}{(\theta-1)^{\lambda-1}}\right)}, \sqrt[q]{\log_\theta\left(1+\frac{(\theta^{\nu^q}-1)^\lambda}{(\theta-1)^{\lambda-1}}\right)} \right)$

(4) $\delta^{\wedge_F \lambda} = \left(\sqrt[q]{\log_\theta\left(1+\frac{(\theta^{\mu^q}-1)^\lambda}{(\theta-1)^{\lambda-1}}\right)}, \sqrt[q]{1-\log_\theta\left(1+\frac{(\theta^{1-\nu^q}-1)^\lambda}{(\theta-1)^{\lambda-1}}\right)} \right)$

It is easy to prove that the above calculation results are still q-ROFNs, which is omitted.

Theorem 1. Suppose δ=(μ, ν), δ1=(μ1, ν1) and δ2=(μ2, ν2) are three q-ROFNs, λ1, λ2, λ ≥ 0, then they have the following operational properties:

(1) $\delta_1 \oplus_F \delta_2 = \delta_2 \oplus_F \delta_1$;

(2) $\delta_1 \otimes_F \delta_2 = \delta_2 \otimes_F \delta_1$;

(3) $\lambda \cdot_F (\delta_1 \oplus_F \delta_2) = \lambda \cdot_F \delta_2 \oplus_F \lambda \cdot_F \delta_1$;

(4) $\lambda_1 \cdot_F \delta \oplus_F \lambda_2 \cdot_F \delta = (\lambda_1+\lambda_2) \cdot_F \delta$;

(5) $\delta^{\wedge_F \lambda_1} \otimes_F \delta^{\wedge_F \lambda_2} = \delta^{\wedge_F (\lambda_1+\lambda_2)}$;

(6) $\delta_1^{\wedge_F \lambda} \otimes_F \delta_2^{\wedge_F \lambda} = (\delta_1 \otimes_F \delta_2)^{\wedge_F \lambda}$.

3.2 The q-ROFFSWA and q-ROFFSWG operations

Definition 8. Suppose δj=(μj, νj) (j =1, 2,…, n) is a family of q-ROFNs, then the q-ROFFSCA and q-ROFFSCG: Ωn→Ω. If

$$q-ROFFSWA(\delta_1,\delta_2,\dots,\delta_n) = \underset{j=1}{\overset{n}{\oplus_F}} \left(\phi_j^\kappa \cdot_F \delta_j\right) \tag{8}$$

$$q-ROFFSWG(\delta_1,\delta_2,\dots,\delta_n) = \underset{j=1}{\overset{n}{\otimes_F}} \left(\delta_j\right)^{\wedge_F \phi_j^\kappa} \tag{9}$$

The $q-ROFFSWG(\delta_1,\delta_2,\dots,\delta_n)$ and $q-ROFFSWA(\delta_1,\delta_2,\dots,\delta_n)$ are called the q-ROF Frank softmax weighted average (q-ROFFSWA) operator and the q-ROF Frank softmax weighted geometric (q-ROFFSWG) operator, respectively. $\phi_j^\kappa = \frac{w_j \exp\left(T_j/\kappa\right)}{\sum_{j=1}^n w_j \exp\left(T_j/\kappa\right)}$ (κ>0) satisfies $\phi_j^\kappa \in [0,1]$ $\sum_{j=1}^n \phi_j^\kappa = 1$.

where $T_j = \begin{cases} \prod_{l=1}^{j-1} Sc(\delta_j) & j=2,3,\dots,n \\ 1 & j=1 \end{cases}$, Sc(δj) is the score function of q-ROFN δj, w=(w1,w2,…,wn)T is the weight vector of δj with $\sum_{j=1}^{n} w_j = 1, w_j \geq 0$.

Theorem 2. Suppose δj (j=1, 2,…, n) is a family of q-ROFNs, the aggregation results of q-ROFFSWA and q-ROFFSWG operators in Definition 8 are still the q-ROFNs.

$$q\text{-}ROFFSWA(\delta_1, \delta_2, \ldots, \delta_n) = \left\langle \sqrt[q]{1 - \log_\theta\left(1 + \prod_{j=1}^{n}\left(\theta^{1-\mu_j^q} - 1\right)^{\frac{w_j \exp(T_j/\kappa)}{\sum_{j=1}^{n} w_j \exp(T_j/\kappa)}}\right)}, \sqrt[q]{\log_\theta\left(1 + \prod_{j=1}^{n}\left(\theta^{\nu_j^q} - 1\right)^{\frac{w_j \exp(T_j/\kappa)}{\sum_{j=1}^{n} w_j \exp(T_j/\kappa)}}\right)} \right\rangle \tag{10}$$

$$q\text{-ROFFSWG}(\delta_1, \delta_2, \ldots, \delta_n) = \left\langle \sqrt[q]{\log_\theta\left(1 + \prod_{j=1}^{n}\left(\theta^{\mu_j^q} - 1\right)^{\frac{w_j \exp(T_j/\kappa)}{\sum_{j=1}^{n} w_j \exp(T_j/\kappa)}}\right)}, \sqrt[q]{1 - \log_\theta\left(1 + \prod_{j=1}^{n}\left(\theta^{1-\nu_j^q} - 1\right)^{\frac{w_j \exp(T_j/\kappa)}{\sum_{j=1}^{n} w_j \exp(T_j/\kappa)}}\right)} \right\rangle \tag{11}$$

Proof: By mathematical induction. According to Definition 7, we have

If n=2, then

$$\phi_1^\kappa = \frac{w_1 \exp(T_1/\kappa)}{\sum_{j=1}^{2} w_j \exp(T_j/\kappa)}, \quad \phi_1^\kappa \cdot_F \delta_1 = \left(\sqrt[q]{1 - \log_\theta\left(1 + \frac{\left(\theta^{1-\mu_1^q} - 1\right)^{\phi_1^\kappa}}{(\theta - 1)^{\phi_1^\kappa - 1}}\right)}, \sqrt[q]{\log_\theta\left(1 + \frac{\left(\theta^{\nu_1^q} - 1\right)^{\phi_1^\kappa}}{(\theta - 1)^{\phi_1^\kappa - 1}}\right)} \right)$$

$$\phi_2^\kappa = \frac{w_2 \exp(T_2/\kappa)}{\sum_{j=1}^{2} w_j \exp(T_j/\kappa)}, \quad \phi_2^\kappa \cdot_F \delta_2 = \left(\sqrt[q]{1 - \log_\theta\left(1 + \frac{\left(\theta^{1-\mu_2^q} - 1\right)^{\phi_2^\kappa}}{(\theta - 1)^{\phi_2^\kappa - 1}}\right)}, \sqrt[q]{\log_\theta\left(1 + \frac{\left(\theta^{\nu_2^q} - 1\right)^{\phi_2^\kappa}}{(\theta - 1)^{\phi_2^\kappa - 1}}\right)} \right),$$

Then,

$$q - ROFFSWA(\delta_1, \delta_2) = \phi_1^\kappa \cdot_F \delta_1 \oplus_F \phi_2^\kappa \cdot_F \delta_2$$

$$= \left(\sqrt[q]{1 - \log_\theta\left(1 + \frac{\left(\theta^{1-\mu_1^q} - 1\right)^{\phi_1^\kappa}}{(\theta - 1)^{\phi_1^\kappa - 1}}\right)}, \sqrt[q]{\log_\theta\left(1 + \frac{\left(\theta^{\nu_1^q} - 1\right)^{\phi_1^\kappa}}{(\theta - 1)^{\phi_1^\kappa - 1}}\right)} \right) \oplus_F \left(\sqrt[q]{1 - \log_\theta\left(1 + \frac{\left(\theta^{1-\mu_2^q} - 1\right)^{\phi_2^\kappa}}{(\theta - 1)^{\phi_2^\kappa - 1}}\right)}, \sqrt[q]{\log_\theta\left(1 + \frac{\left(\theta^{\nu_2^q} - 1\right)^{\phi_2^\kappa}}{(\theta - 1)^{\phi_2^\kappa - 1}}\right)} \right)$$

$$= \left(\sqrt[q]{1 - \log_\theta\left(1 + \frac{\prod_{j=1}^{2}\left(\theta^{1-\mu_j^q} - 1\right)^{\phi_j^\kappa}}{(\theta - 1)^{\sum_{j=1}^{2}\phi_j^\kappa - 1}}\right)}, \sqrt[q]{\log_\theta\left(1 + \frac{\prod_{j=1}^{2}\left(\theta^{\nu_j^q} - 1\right)^{\phi_j^\kappa}}{(\theta - 1)^{\sum_{j=1}^{2}\phi_j^\kappa - 1}}\right)} \right)$$

Since $\sum_{j=1}^{2} \phi_j^\kappa = 1$, then we have

$$q - ROFFSWA(\delta_1, \delta_2) = \left(\sqrt[q]{1 - \log_\theta\left(1 + \prod_{j=1}^{2}\left(\theta^{1-\mu_j^q} - 1\right)^{\phi_j^\kappa}\right)}, \sqrt[q]{\log_\theta\left(1 + \prod_{j=1}^{2}\left(\theta^{\nu_j^q} - 1\right)^{\phi_j^\kappa}\right)} \right).$$

This means that Eq.(10) holds for n=2.

If n=m, the Eq.(10) holds, namely,

$$q - ROFFSWA(\delta_1, \delta_2, \ldots, \delta_m) = \left(\sqrt[q]{1 - \log_\theta\left(1 + \prod_{j=1}^{m}\left(\theta^{1-\mu_j^q} - 1\right)^{\phi_j^\kappa}\right)}, \sqrt[q]{\log_\theta\left(1 + \prod_{j=1}^{m}\left(\theta^{\nu_j^q} - 1\right)^{\phi_j^\kappa}\right)} \right)$$

If n=m+1, we have

$$q-ROFFSWA(\delta_1,\delta_2,\ldots,\delta_m,\delta_{m+1}) = q-ROFFSWA(\delta_1,\delta_2,\ldots,\delta_m) \oplus_F \phi_{m+1}^{\kappa} \cdot_F \delta_{m+1}$$

$$= \left(\sqrt[q]{1-\log_\theta\left(1+\prod_{j=1}^{m}\left(\theta^{1-\mu_j^q}-1\right)^{\phi_j^\kappa}\right)}, \sqrt[q]{\log_\theta\left(1+\prod_{j=1}^{m}\left(\theta^{\nu_j^q}-1\right)^{\phi_j^\kappa}\right)} \right) \oplus_F \left(\sqrt[q]{1-\log_\theta\left(1+\frac{\left(\theta^{1-\mu_{m+1}^q}-1\right)^{\phi_{m+1}^\kappa}}{(\theta-1)^{\phi_{m+1}^\kappa-1}}\right)}, \sqrt[q]{\log_\theta\left(1+\frac{\left(\theta^{\nu_{m+1}^q}-1\right)^{\phi_{m+1}^\kappa}}{(\theta-1)^{\phi_{m+1}^\kappa-1}}\right)} \right)$$

$$= \left(\sqrt[q]{1-\log_\theta\left(1+\frac{\prod_{j=1}^{m+1}\left(\theta^{1-\mu_j^q}-1\right)^{\phi_j^\kappa}}{(\theta-1)^{\sum_{j=1}^{m+1}\phi_j^\kappa-1}}\right)}, \sqrt[q]{\log_\theta\left(1+\frac{\prod_{j=1}^{m+1}\left(\theta^{\nu_j^q}-1\right)^{\phi_j^\kappa}}{(\theta-1)^{\sum_{j=1}^{m+1}\phi_j^\kappa-1}}\right)} \right)$$

Since $\sum_{j=1}^{m+1}\phi_j^\kappa = 1$, then we have

$$q-ROFFSWA(\delta_1,\delta_2,\ldots,\delta_{m+1}) = \left(\sqrt[q]{1-\log_\theta\left(1+\prod_{j=1}^{m+1}\left(\theta^{1-\mu_j^q}-1\right)^{\phi_j^\kappa}\right)}, \sqrt[q]{\log_\theta\left(1+\prod_{j=1}^{m+1}\left(\theta^{\nu_j^q}-1\right)^{\phi_j^\kappa}\right)} \right)$$

So, the Eq. (10) holds for n=m+1.

Thus, the Eq. (10) holds for all n. Similarly, the Eq. (11) can also be proved to hold for all n.

Therefore, we complete the proof of Theorem 2.

The following desirable properties of the q-ROFFSWA and q-ROFFSWG operators can be proved easily:

Theorem 3 (Idempotency). Suppose δj (j=1, 2,…, n) is a family of q-ROFNs, if δj = δ, then

$$q\text{-}ROFFSWA(\delta_1,\delta_2,\ldots,\delta_n) = q\text{-}ROFFSWG(\delta_1,\delta_2,\ldots,\delta_n) = p \quad (12)$$

Theorem 4 (Boundedness). Suppose δj (j=1, 2,…, n) is a family of q-ROFNs, if $P^- = \min\delta_j = (\min_j(\mu_j), \max_j(\nu_j))$, $P^+ = \max\delta_j = (\max_j(\mu_j), \min_j(\nu_j))$, then

$$P^- \le q\text{-}ROFFSWA(\delta_1,\delta_2,\ldots,\delta_n) \le P^+ \quad (13)$$

$$P^- \le q\text{-}ROFFSWG(\delta_1,\delta_2,\ldots,\delta_n) \le P^+ \quad (14)$$

Theorem 5 (Monotonicity). Suppose δj and δ*j = (μ*j, ν*j) (j=1, 2,…, n) are two families of q-ROFNs, if δi ≤δi*, then

$$q\text{-}ROFFSWA(\delta_1,\delta_2,\ldots,\delta_n) \le q\text{-}ROFFSWA(\delta_1^*,\delta_2^*,\ldots,\delta_n^*) \quad (15)$$

$$q\text{-}ROFFSWG(\delta_1,\delta_2,\ldots,\delta_n) \le q\text{-}ROFFSWG(\delta_1^*,\delta_2^*,\ldots,\delta_n^*) \quad (16)$$

3.3 The family analysis of proposed AOs

We can obtain the following several particular cases by giving various values of parameters q, θ and κ :

Theorem 6. Suppose δj (j=1, 2,…, n) is a family of q-ROFNs, then

(1) If q=1, θ→1, then the q-ROFFSWA and q-ROFFSWG operators are reduced to the softmax intuitionistic fuzzy weight average (SIFWA) and softmax intuitionistic fuzzy weighted geometric (SIFWG) operators [39], respectively, i.e.,

$$\lim_{\theta\to1} q-ROFFSWA_{q=1}(\delta_1,\delta_2,\ldots,\delta_n) = SIFWA(\delta_1,\delta_2,\ldots,\delta_n) = \left(1-\prod_{j=1}^{n}\left(1-\mu_j\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}, \prod_{j=1}^{n}\left(\nu_j\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right) \quad (17)$$

$$\lim_{\theta\to1} q-ROFFSWG_{q=1}(\delta_1,\delta_2,\ldots,\delta_n) = SIFWG(\delta_1,\delta_2,\ldots,\delta_n) = \left(\prod_{j=1}^{n}\left(\mu_j\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}, 1-\prod_{j=1}^{n}\left(1-\nu_j\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right) \quad (18)$$

(2) If q=2, θ→1, then the q-ROFFSWA and q-ROFFSWG operators are reduced to the softmax Pythagorean fuzzy weighted averaging (SPFWA) and softmax Pythagorean fuzzy weighted geometric (SPFWG) operators, respectively, i.e.,

$$\lim_{\theta\to 1} q-ROFFSWA_{q=2}(\delta_1,\delta_2,\ldots,\delta_n)=SPFWA(\delta_1,\delta_2,\ldots,\delta_n)=\left(\sqrt{1-\prod_{j=1}^{n}\left(1-\mu_j^2\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}},\prod_{j=1}^{n}\left(\nu_j\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right) \quad (19)$$

$$\lim_{\theta\to 1} q-ROFFSWG_{q=2}(\delta_1,\delta_2,\ldots,\delta_n)=SPFWG(\delta_1,\delta_2,\ldots,\delta_n)=\left(\prod_{j=1}^{n}\left(\mu_j\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}},\sqrt{1-\prod_{j=1}^{n}\left(1-\nu_j^2\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}}\right) \quad (20)$$

(3) If q=1, κ→+∞, θ→1, then the q-ROFFSWA and q-ROFFSWG operators are reduced to the intuitionistic fuzzy weighted average (IFWA) and intuitionistic fuzzy weighted geometric (IFWG) operators [47], respectively, i.e.,

$$\lim_{\substack{\theta\to 1\\ \kappa\to+\infty}} q-ROFFSWA_{q=1}(\delta_1,\delta_2,\ldots,\delta_n)=IFWA(\delta_1,\delta_2,\ldots,\delta_n)=\left(1-\prod_{j=1}^{n}\left(1-\mu_j\right)^{w_j},\prod_{j=1}^{n}\left(\nu_j\right)^{w_j}\right) \quad (21)$$

$$\lim_{\substack{\theta\to 1\\ \kappa\to+\infty}} q-ROFFSWG_{q=1}(\delta_1,\delta_2,\ldots,\delta_n)=IFWG(\delta_1,\delta_2,\ldots,\delta_n)=\left(\prod_{j=1}^{n}\left(\mu_j\right)^{w_j},1-\prod_{j=1}^{n}\left(1-\nu_j\right)^{w_j}\right) \quad (22)$$

(4) If q=2, κ→+∞, θ→1, then the q-ROFFSWA and q-ROFFSWG operators are reduced to the PyFWA and Pythagorean fuzzy weighted geometric (PyFWG) operators [48], respectively, i.e.,

$$\lim_{\substack{\theta\to 1\\ \kappa\to+\infty}} q-ROFFSWA_{q=2}(\delta_1,\delta_2,\ldots,\delta_n)=PFWA(\delta_1,\delta_2,\ldots,\delta_n)=\left(\sqrt{1-\prod_{j=1}^{n}\left(1-\mu_j^2\right)^{w_j}},\prod_{j=1}^{n}\left(\nu_j\right)^{w_j}\right) \quad (23)$$

$$\lim_{\substack{\theta\to 1\\ \kappa\to+\infty}} q-ROFFSWG_{q=2}(\delta_1,\delta_2,\ldots,\delta_n)=PFWG(\delta_1,\delta_2,\ldots,\delta_n)=\left(\prod_{j=1}^{n}\left(\mu_j\right)^{w_j},\sqrt{1-\prod_{j=1}^{n}\left(1-\nu_j^2\right)^{w_j}}\right) \quad (24)$$

(5) If q=1, κ→+∞, θ→+∞, then the q-ROFFSWA and q-ROFFSWG operators are reduced to the traditional arithmetic weighted mean operator, i.e.,

$$\lim_{\substack{\theta\to+\infty\\ \kappa\to+\infty}} q-ROFFSWA_{q=1}(\delta_1,\delta_2,\ldots,\delta_n)=\lim_{\substack{\theta\to+\infty\\ \kappa\to+\infty}} q-ROFFSWG_{q=1}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\sum_{j=1}^{n}w_j\mu_j,\sum_{j=1}^{n}w_j\nu_j\right) \quad (25)$$

(6) If q=2, κ→+∞, θ→+∞, then the q-ROFFSWA and q-ROFFSWG operators are reduced to the traditional arithmetic weighted mean operator, i.e.,

$$\lim_{\substack{\theta\to+\infty\\ \kappa\to+\infty}} q-ROFFSWA_{q=2}(\delta_1,\delta_2,\ldots,\delta_n)=\lim_{\substack{\theta\to+\infty\\ \kappa\to+\infty}} q-ROFFSWG_{q=2}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\sqrt{\sum_{j=1}^{n}w_j\mu_j^2},\sqrt{\sum_{j=1}^{n}w_j\nu_j^2}\right) \quad (26)$$

3.4 Monotonicity analysis on parameter ϑ

Theorem 7. Suppose δj (j=1, 2,…,n) is a family of q-ROFNs, then the score function of the aggregation result calculated by q-ROFFSWA operator decreases monotonically with θ, while the score function of the aggregation result calculated by q-ROFFSWG operator increases monotonically with θ.

Proof: We first prove that the score function of the aggregation result calculated by q-ROFFSWA operator decreases monotonically with θ. From Definition 2, the score function of Eq. (10) can be obtained

$$g(\theta)=Sc\left(q-ROFFSWA(p_1,p_2,\cdots,p_n)\right)$$

$$=1-\frac{1}{2}\left(\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)+\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\right)$$

Let $f(\theta)=\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)+\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)$, we take the first-order derivative of f(θ) with respect to θ, then we have

$$\frac{df(\theta)}{d\theta}=\frac{\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\sum_{j=1}^{n}\frac{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}\left(\theta^{1-\mu_j^q}-1\right)\theta^{-\mu_j^q}}{\theta^{\mu_j^q}-1}}{\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\ln\theta}+\frac{\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\sum_{j=1}^{n}\frac{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}\left(\theta^{\nu_j^q}-1\right)\theta^{\nu_j^q-1}}{\theta^{1-\nu_j^q}-1}}{\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\ln\theta}$$

Since θ>1, 0 ≤ μj, νj ≤1, $0\le\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}\le 1$, then $\frac{df(\theta)}{d\theta}>0$. It can be obtained that f(θ) increases monotonically with θ. Then g(θ)=1-0.5×f(θ), so, the g(θ) decreases monotonically with θ, namely, the score function of the aggregation result calculated by the q-ROFFSWA operator decreases monotonically with θ. In the same way, the score function of q-ROFFSWG operator increases monotonically with θ can also be proved to be true. Therefore, the Theorem 7 holds.

Theorem 8. If δj (j=1, 2,…, n) is a family of q-ROFNs, then the q-ROFFSWA operator is greater than or equal to the q-ROFFSWG operator, i.e., q-ROFFSWA (δ1, δ2,…, δn) ≥ q-ROFFSWG (δ1, δ2,…, δn), (θ>1, κ>0, q ≥1).

Proof: Let the score function of q-ROFFSWA (δ1, δ2,…, δn) be Sc(A) and the score function of q-ROFFSWG (δ1, δ2,…, δn) be Sc(G). According to Eq.(3), we have

$$Sc(A)=\frac{1}{2}\left(2-\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)-\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\right)$$

$$Sc(G)=\frac{1}{2}\left(\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)+\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{1-\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\right)$$

then,

$$Sc(A)-Sc(G)$$

$$=\frac{1}{2}\left(2-\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)-\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\right)$$

$$-\frac{1}{2}\left(\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{\mu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)+\log_\theta\left(1+\prod_{j=1}^{n}\left(\theta^{1-\nu_j^q}-1\right)^{\frac{w_j\exp\left(T_j/\kappa\right)}{\sum_{j=1}^{n}w_j\exp\left(T_j/\kappa\right)}}\right)\right)$$

$$=\frac{1}{2}\left(\begin{array}{l}2-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)\\-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{\mu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{1-\nu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)\end{array}\right)$$

$$=\frac{1}{2}\left\{\begin{array}{l}\left(1-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)\right)\\+\left(1-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{\mu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)-\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{1-\nu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)\right)\end{array}\right\}$$

According to Theorem 2, we have

$$\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{1-\mu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)+\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{\nu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)\le 1\text{ ,and}$$

$$\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{\mu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)+\log_{\theta}\left(1+\prod_{j=1}^{n}\left(\theta^{1-\nu_j^q}-1\right)^{\frac{w_j\exp(T_j/\kappa)}{\sum_{j=1}^{n}w_j\exp(T_j/\kappa)}}\right)\le 1\,.$$

So, we can obtain $Sc(A)-Sc(G)\ge 0$.

Therefore, we have proved that q-ROFFSWA (δ1, δ2,…,δn) ≥ q-ROFFSWG (δ1, δ2,…,δn).

From the monotonicity in the Theorem 7, we can find that different values of parameter θ contained in the AOs proposed in this paper can indicate the type and degree of DMs' risk preference. The score function of q-ROFFSWA operator shows a decreasing trend in the range of parameter θ values. When the DM is risk-averse, a larger parameter value is taken. When the DM is risk-seeking, the smaller parameter value is taken. However, the q-ROFFSWG operator is the opposite.In addition, the size relationship between q-ROFFSWA and q-ROFFSWG in Theorem 8 shows that the q-ROFFSWA operator has a higher comprehensive evaluation value and is suitable for optimistic DMs. The parameter θ also can describe the level of optimism of DMs, while the q-ROFFSWG operator is suitable for pessimistic DMs. The parameter θ can describe the level of pessimism of DMs. Therefore, the individual assessment information is fused by the q-ROFFSWA operator, and aggregation the result can indicate that DMs with optimistic decision attitude can flexibly adjust the type of risk preference (seeking or averse) through parameter θ. But the q-ROFFSWG operator has the opposite meaning.

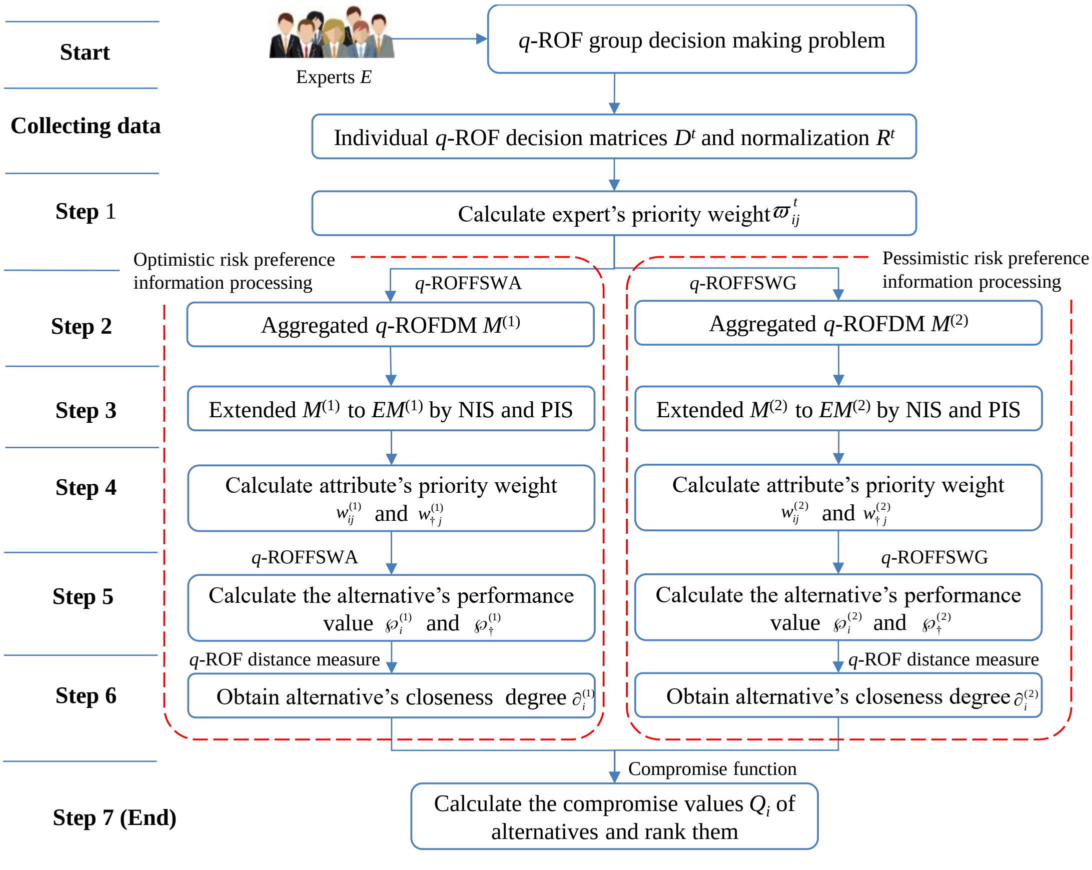
4 An improved WASPAS for MAGDM problems with q-ROFNs

There are m alternatives si (i=1, 2,…,m) to form the alternative set as S={s1, s2, …, sm}. The attribute set is composed of n attributes hj (j=1, 2,…, n), which is H={h1, h2,…, hn}, and ω=(ω1, ω2,…, ωn)T is the weight vector of attribute set H, satisfies $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. The expert set E={e1, e2,…, ez} is composed of z experts et (t=1, 2,…, z), λ=(λ1,λ2,…,λz)T is the weight vector of expert set E, and satisfies $\lambda_t\in[0,1]$, $\sum_{t=1}^{z}\lambda_t=1$. Experts evaluate alternative si (i=1, 2,…,m) according to attribute hj (j=1, 2,…, n), and then the individual q-ROFdecision matrix (q-ROFDM) of expert et is

$D^t = \left[d_{ij}^t\right]_{m\times n}$, $d_{ij}^t = (\mu_{ij}^t, \nu_{ij}^t)$ (i=1, 2,..., m; j=1, 2,..., n; t=1, 2,..., z), meeting $0 \le \mu_{ij}^t, \nu_{ij}^t \le 1$ and $(\mu_{ij}^t)^q + (\nu_{ij}^t)^q \le 1$ (q≥1). Generally, we usually need to normalize Dt and transform it from Eq.(27) to obtain the normalized q-ROFDM Rt=[rijt]m×n (i=1, 2,..., m; j=1, 2,..., n; t=1, 2,..., z).

$$r_{ij}^t = \begin{cases} d_{ij}^t = \left(\mu_{ij}^t, \nu_{ij}^t\right) & , \; c_j \in \Psi_1 \\ (d_{ij}^t)^c = \left(\mu_{ij}^t, \nu_{ij}^t\right), & c_j \in \Psi_2 \end{cases} \tag{27}$$

where $(d_{ij}^t)^c$ is the complement set of q-ROFN d_{ij}^t, Ψ1 and Ψ2 represent benefit and cost attributes respectively.



. 1. The flowchart of improved WASPAS method

According to the characteristics of decision attitude with risk preference of the q-ROFFSWA and q-ROFFSWG operators in sub-section 3.4, we establish two independent and parallel calculation processes, namely the optimistic and pessimistic risk preference information processing (see Figure 1). We present the following detailed algorithm procedure.

Step 1: Calculate the priority weight ϖ_{ij}^t of experts with Eq. (28).

$$\varpi_{ij}^t = \frac{\lambda_t \exp\left(T_{ij}^t / \kappa\right)}{\sum_{t=1}^{z} \lambda_t \exp\left(T_{ij}^t / \kappa\right)} \tag{28}$$

where $T_{ij}^t = \begin{cases} \prod_{h=1}^{t-1} Sc(r_{ij}^t), & t = 2,3,\ldots,z \\ 1, & t = 1 \end{cases}$, $\lambda_t$is the expert weight and κ is the modulation parameter, κ> 0.

Step 2: We adopt the q-ROFFSWA and q-ROFFSWG operators to fuse the individual assessment information from experts in the q-ROFDM Rt, and then form the group q-ROFDM $M^{(1)}=\left[g_{ij}^{(1)}\right]_{m\times n}$ and $M^{(2)}=\left[g_{ij}^{(2)}\right]_{m\times n}$, respectively.

$$g_{ij}^{(1)}=q-ROFFSWA(r_{ij}^{1},r_{ij}^{2},\ldots,r_{ij}^{z})=\left(\sqrt[q]{1-\log_{\theta}\left(1+\prod_{t=1}^{z}\left(\theta^{1-(\mu_{ij}^{t})^{q}}-1\right)^{\varpi_{ij}^{t}}\right)},\sqrt[q]{\log_{\theta}\left(1+\prod_{i=1}^{n}\left(\theta^{(\nu_{ij}^{t})^{q}}-1\right)^{\varpi_{ij}^{t}}\right)}\right) \tag{29}$$

$$g_{ij}^{(2)}=q-ROFFSWG(r_{ij}^{t},r_{ij}^{t},\ldots,r_{ij}^{t})=\left(\sqrt[q]{\log_{\theta}\left(1+\prod_{t=1}^{z}\left(\theta^{(\mu_{ij}^{t})^{q}}-1\right)^{\varpi_{ij}^{t}}\right)},\sqrt[q]{1-\log_{\theta}\left(1+\prod_{t=1}^{z}\left(\theta^{1-(\nu_{ij}^{t})^{q}}-1\right)^{\varpi_{ij}^{t}}\right)}\right) \tag{30}$$

Step 3: Construct the extended group q-ROFDM EM(1) and EM(2).

$$EM^{\Upsilon}=\begin{array}{c} \\ NIS \\ P_1 \\ \vdots \\ P_m \\ PIS \end{array}\begin{array}{c} \begin{array}{cccc} Y_1, & Y_2, & \cdots, & Y_n \end{array} \\ \begin{bmatrix} g_1^{\Upsilon\,NIS} & g_2^{\Upsilon\,NIS} & \cdots & g_n^{\Upsilon\,NIS} \\ g_{11}^{\Upsilon} & g_{12}^{\Upsilon} & \cdots & g_{1n}^{\Upsilon} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1}^{\Upsilon} & g_{m2}^{\Upsilon} & \cdots & g_{mn}^{\Upsilon} \\ g_1^{\Upsilon\,PIS} & g_2^{\Upsilon\,PIS} & \cdots & g_n^{\Upsilon\,PIS} \end{bmatrix} \end{array} \tag{31}$$

where Υ represent the (1) and (2), the NIS and PIS mean the negative ideal solution (NIS) and positive ideal solution (PIS) of the group q-ROFDM Mϒ, respectively, that is $g_j^{\Upsilon\,NIS}=\left(\min_i(\mu_{ij}),\max_i(\nu_{ij})\right)$ and $g_j^{\Upsilon\,PIS}=\left(\max_i(\mu_{ij}),\min_i(\nu_{ij})\right)$.

Step 4: Calculate the T_{ij}^{Υ} and $T_{\dagger j}^{\Upsilon}$ with Eq. (32), and then calculate attribute priority weight values w_{ij}^{Υ}, $w_{\dagger j}^{\Upsilon}$ with Eq. (33).

$$T_{ij}^{\Upsilon}=\begin{cases}\prod_{l=1}^{j-1} sc(g_{ij\,ij}^{\Upsilon}),\ j=2,3,\ldots,n \\ 1\quad,\ j=1\end{cases};T_{\dagger j}^{\Upsilon}=\begin{cases}\prod_{l=1}^{j-1} sc(g_{\dagger j}^{\Upsilon}),\ j=2,3,\ldots,n \\ 1\quad,\ j=1\end{cases} \tag{32}$$

$$w_{ij}^{\Upsilon}=\frac{\omega_j\exp\left(T_{ij}^{\Upsilon}/\kappa\right)}{\sum_{j=1}^{n}\omega_j\exp\left(T_{ij}^{\Upsilon}/\kappa\right)}\ ;\ w_{\dagger j}^{\Upsilon}=\frac{\omega_j\exp\left(T_j^{\dagger}/\kappa\right)}{\sum_{j=1}^{n}\omega_j\exp\left(T_j^{\dagger}/\kappa\right)} \tag{33}$$

where Υ represent the (1) and (2), and "†"denotes NIS and PIS respectively.

Step 5: The q-ROFNs of all attributes cj in the extended group q-ROFDM are aggregated by Eq. (34-35) to obtain the performance values of all alternatives.

$$\begin{cases}\wp_i^{(1)}=q-ROFFSWA(g_{i1}^{(1)},g_{i2}^{(1)},\ldots,g_{in}^{(1)}) \\ \wp_{\dagger}^{(1)}=q-ROFFSWA(g_{\dagger 1}^{(1)},g_{\dagger 2}^{(1)},\ldots,g_{\dagger n}^{(1)})\end{cases} \tag{34}$$

and

$$\begin{cases}\wp_i^{(2)}=q-ROFFWG(g_{i1}^{(2)},g_{i2}^{(2)},\ldots,g_{in}^{(2)}) \\ \wp_{\dagger}^{(2)}=q-ROFFWG(g_{\dagger 1}^{(2)},g_{\dagger 2}^{(2)},\ldots,g_{\dagger n}^{(2)})\end{cases} \tag{35}$$

where "†"denotes NIS and PIS respectively.

Step 6: Combined with the q-ROF distance measure, the Eqs. (36-37) are used to calculate the closeness degree $\partial_i^{(1)}$ and $\partial_i^{(2)}$.

$$\partial_i^{(1)}=\frac{D_H(\wp_i^{(1)},\wp_{NIS}^{(1)})}{D_H(\wp_i^{(1)},\wp_{NIS}^{(1)})+D_H(\wp_i^{(1)},\wp_{PIS}^{(1)})} \tag{36}$$

$$\partial_i^{(2)} = \frac{D_H(\wp_i^{(2)}, \wp_{NIS}^{(2)})}{D_H(\wp_i^{(2)}, \wp_{NIS}^{(2)}) + D_H(\wp_i^{(2)}, \wp_{PIS}^{(2)})} \tag{37}$$

In Eq. (36), $D_H(\wp_i^{(1)}, \wp_{NIS}^{(1)})$ represents the q-ROF Hamming distance measure between $\wp_i^{(1)}$ and $\wp_{NIS}^{(1)}$. The closeness degree $\partial_i^{(1)}$ and $\partial_i^{(2)}$ are the results of information processing of experts' optimistic and pessimistic decision attitudes with risk preference. To balance these two opposite decision attitudes, we construct a compromise function, that is, the calculation of the compromise value of alternative ai, as shown in Eq. (38).

$$Q_i = \rho \frac{\partial_i^{(1)} - \min_i \partial_i^{(1)}}{\max_i \partial_i^{(1)} - \min_i \partial_i^{(1)}} + (1-\rho) \frac{\partial_i^{(2)} - \min_i \partial_i^{(2)}}{\max_i \partial_i^{(2)} - \min_i \partial_i^{(2)}} \tag{38}$$

where ρ is a compromise coefficient, ρ∈[0,1]. If the smaller compromise coefficient (ρ∈[0,0.5)) is selected, it means that the decision is made according to the optimistic decision attitude of the DMs. If the larger compromise coefficient (ρ∈(0.5,1]) is taken, it means to made a decision based on pessimistic decision attitude of the DMs. This article takes ρ=0.5 to determine the best option in a neutral way, which means DMs have reached a consensus through consultation.

Step 7: The compromise value Qi (i=1, 2,…, m) of each alternative is computed by Eq. (38), and the final compromise ranking of the alternatives is determine, that is, the biggest value of Qi is the best option.

5 Case study: Investment decision of CGB platform

The CGB is a kind of shopping and consumption behavior with low discount that a certain number of consumer groups in real living communities purchase goods online and pick up goods offline. Compared with online group-buying, the CGB requires the establishment of a service center in the community or other specific places, where consumers can pay the money and obtain after-sales protection in case of problems with commodities. The CGB has the characteristics of regionalization, minority, localization and individuation, which is favored by the majority of community residents.

Currently, there are many CGB platforms in the group-buying market in China. Due to the fierce competition in the group-buying market, the investment and financing funds obtained by the CGB platform from the capital market increased from RMB 2.402 billion in 2016 to RMB 20.073 billion in 2020. However, for investors, how to select a potential CGB platform as the investment object has become a challenging decision-making problem.

Table 2
Linguistic terms and corresponding q-ROFNs

Linguistic terms	q-ROFNs
EH: Extremely high	(0.950,0.200)
VH: Very high	(0.900,0.400)
H: High	(0.800,0.500)
MH: Medium high	(0.750,0.600)
M: Medium	(0.600,0.700)
ML: Medium low	(0.500,0.800)
L: Low	(0.400,0.850)
VL: Very low	(0.300,0.900)
EL: Extremely low	(0.200,0.950)

LC is a venture capital company and wants to invest in CGB platform. According to the market survey and preliminary screening, there are five CGB platforms (s1, s2, s3, s4, s5) as potential investment objects. In order to screen out the best CGB platform project, LC company invited three senior investment experts E={e1,e2,e3}, suppose the weight of experts is equal and the priority relationship of experts is e1>e2>e3. There are six attributes used to evaluate the alternatives, including: platform operation and maintenance ability (h1), expected revenue (h2), market competitiveness (h3), risk resistance ability (h4), supply chain management ability (h5) and product and service innovation ability (h6). Suppose the attribute weight vector is w=(0.18, 0.20, 0.10, 0.12, 0.25, 0.15)T, and their priority is h2>h5>h1>h6>h4>h3. In order to assess the five CGB platforms with six attributes, the experts use the linguistic grade terms in Table 2 and the evaluation results are shown in Table 3.

Table 3
The evaluation results of experts

Experts(E)	Platforms(S)	h1	h2	h3	h4	h5	h6
e1	s1	MH	H	MH	ML	VH	MH
	s2	H	L	H	L	H	MH
	s3	ML	L	MH	L	M	H
	s4	M	H	M	H	M	ML
	s5	H	H	VL	H	ML	M
e2	s1	H	MH	VH	ML	EL	L
	s2	L	MH	VH	ML	L	H
	s3	L	L	H	A	L	ML
	s4	ML	H	M	L	ML	L
	s5	M	H	M	L	MH	H
e3	s1	H	H	MH	L	ML	L
	s2	H	L	L	H	VL	M
	s3	M	MH	H	L	ML	M
	s4	M	M	L	M	H	H
	s5	L	M	L	MH	L	MH

Thus, we can obtain the q-ROFDM Dt. Since all attributes are benefit type, the q-ROFDM Dt does not need to be normalized, that is, Dt=Rt. The q-ROFDM Rt is shown in Table 4.

Table 4
The q-ROFDM Rt

S	h1	h2	h3	h4	h5	h6
s1	(0.750,0.600), (0.800,0.500), (0.800,0.500)	(0.800,0.500), (0.750,0.600), (0.800,0.500)	(0.750,0.600), (0.900,0.400), (0.750,0.600)	(0.500,0.800), (0.500,0.800), (0.400,0.850)	(0.900,0.400), (0.200,0.950), (0.300,0.950)	(0.750,0.600), (0.400,0.850), (0.400,0.850)
s2	(0.800,0.500), (0.400,0.850), (0.800,0.500)	(0.400,0.850), (0.750,0.600), (0.800,0.500)	(0.800,0.500), (0.900,0.400), (0.400,0.850)	(0.400,0.850), (0.500,0.800), (0.400,0.850)	(0.800,0.500), (0.400,0.850), (0.300,0.900)	(0.750,0.600), (0.800,0.500), (0.600,0.700)
s3	(0.800,0.500), (0.400,0.850), (0.750,0.600)	(0.400,0.850), (0.400,0.850), (0.750,0.600)	(0.750,0.600), (0.800,0.500), (0.800,0.500),	(0.400,0.850), (0.600,0.700), (0.400,0.850)	(0.600,0.700), (0.400,0.850), (0.500,0.800)	(0.800,0.500), (0.500,0.800), (0.600,0.700)
s4	(0.600,0.700), (0.500,0.800), (0.600,0.700)	(0.800,0.500), (0.800,0.500), (0.600,0.700)	(0.600,0.700), (0.600,0.700), (0.400,0.850)	(0.800,0.500), (0.400,0.850), (0.600,0.700)	(0.600,0.700), (0.500,0.800), (0.800,0.500)	(0.500,0.800), (0.400,0.850), (0.800,0.500)
s5	(0.800,0.500), (0.600,0.700), (0.400,0.850)	(0.800,0.500), (0.800,0.500), (0.600,0.700)	(0.300,0.900), (0.600,0.700), (0.400,0.850)	(0.800,0.500), (0.400,0.850), (0.750,0.600)	(0.500,0.800), (0.750,0.600), (0.400,0.850)	(0.600,0.700), (0.800,0.500), (0.750,0.600)

5.1 Decision process

Step 1: According to the data in Table 4, we select parameter q=3 and use Eq.(28) to calculate the priority weight of expert (κ=1), as shown in Table 5.

Step 2-3: the Eqs.(29-30) are utilized to aggregate individual expert evaluation information (θ=2), and the group q-ROFDM M(1) and M(2) are obtained. And they are extended to the q-ROFDM EM(1) and EM(2), as shown in Table 6.

Step 4: Use Eqs. (32-33) to calculate the attribute priority weight value w_{ij}^{Υ} and w_{+j}^{Υ} (κ=1). See Table 7.

Table 5

The priority weight of expert ϖ_{it}^{j}

S	h1			h2			h3		
	e1	e2	e3	e1	e2	e3	e1	e2	e3
s1	0.488	0.273	0.240	0.488	0.273	0.240	0.475	0.289	0.236
s2	0.553	0.228	0.220	0.548	0.231	0.221	0.482	0.316	0.202
s3	0.545	0.234	0.220	0.566	0.219	0.215	0.488	0.273	0.240
s4	0.552	0.232	0.215	0.488	0.290	0.221	0.547	0.243	0.210
s5	0.529	0.263	0.208	0.488	0.290	0.221	0.566	0.222	0.211
S	h4			h5			h6		
	e1	e2	e3	e1	e2	e3	e1	e2	e3
s1	0.562	0.227	0.211	0.568	0.222	0.210	0.555	0.234	0.211
s2	0.566	0.223	0.211	0.554	0.238	0.208	0.500	0.279	0.221
s3	0.561	0.228	0.211	0.560	0.227	0.212	0.538	0.245	0.217
s4	0.548	0.236	0.216	0.548	0.231	0.221	0.562	0.221	0.217
s5	0.545	0.234	0.220	0.548	0.242	0.210	0.516	0.257	0.228

Table 6

The group q-ROFDM EM(1) and EM(2)

EM(1)	h1	h2	h3	h4	h5	h6
NIS(1)	(0.580,0.723)	(0.534,0.792)	(0.430,0.843)	(0.427,0.839)	(0.547,0.758)	(0.596,0.737)
s1	(0.777,0.547)	(0.788,0.526)	(0.810,0.535)	(0.483,0.810)	(0.799,0.596)	(0.653,0.704)
s2	(0.597,0.568)	(0.645,0.703)	(0.809,0.523)	(0.427,0.839)	(0.694,0.649)	(0.742,0.591)
s3	(0.740,0.593)	(0.534,0.792)	(0.777,0.547)	(0.465,0.815)	(0.547,0.754)	(0.719,0.607)
s4	(0.580,0.723)	(0.770,0.540)	(0.571,0.730)	(0.714,0.614)	(0.648,0.673)	(0.596,0.737)
s5	(0.712,0.614)	(0.770,0.540)	(0.430,0.843)	(0.740,0.593)	(0.579,0.758)	(0.704,0.621)
PIS(1)	(0.777,0.547)	(0.788,0.526)	(0.810,0.523)	(0.740,0.593)	(0.799,0.596)	(0.742,0.591)
EM(2)	h1	h2	h3	h4	h5	h6
NIS(2)	(0.530,0.728)	(0.460,0.818)	(0.373,0.862)	(0.421,0.840)	(0.527,0.824)	(0.530,0.778)
s1	(0.775,0.554)	(0.786,0.532)	(0.792,0.558)	(0.477,0.812)	(0.529,0.824)	(0.572,0.750)
s2	(0.530,0.642)	(0.545,0.765)	(0.731,0.616)	(0.421,0.840)	(0.562,0.751)	(0.728,0.605)
s3	(0.677,0.660)	(0.460,0.818)	(0.775,0.554)	(0.439,0.826)	(0.527,0.767)	(0.674,0.656)
s4	(0.575,0.728)	(0.753,0.562)	(0.552,0.743)	(0.644,0.680)	(0.615,0.700)	(0.530,0.778)
s5	(0.648,0.676)	(0.753,0.562)	(0.373,0.862)	(0.677,0.660)	(0.529,0.781)	(0.682,0.640)
PIS(2)	(0.775,0.554)	(0.786,0.532)	(0.792,0.554)	(0.677,0.660)	(0.615,0.700)	(0.728,0.605)

Table 7
Attribute priority weight

	h1		h2		h3		h4		h5		h6	
	$w^{(1)}$	$w^{(2)}$	$w^{(1)}$	$w^{(2)}$	$w^{(1)}$	$w^{(2)}$	$w^{(1)}$	$w^{(2)}$	$w^{(1)}$	$w^{(2)}$	$w^{(1)}$	$w^{(2)}$
NIS	0.136	0.135	0.392	0.396	0.072	0.073	0.087	0.088	0.203	0.198	0.110	0.110
s1	0.153	0.143	0.347	0.380	0.065	0.070	0.080	0.085	0.247	0.212	0.109	0.110
s2	0.141	0.137	0.374	0.389	0.070	0.072	0.084	0.087	0.220	0.205	0.111	0.111
s3	0.138	0.137	0.389	0.393	0.072	0.073	0.087	0.087	0.202	0.199	0.112	0.111
s4	0.139	0.138	0.369	0.374	0.069	0.069	0.084	0.084	0.232	0.227	0.107	0.107
s5	0.142	0.139	0.373	0.379	0.069	0.070	0.086	0.086	0.219	0.215	0.111	0.110
PIS	0.151	0.146	0.342	0.363	0.068	0.069	0.084	0.085	0.244	0.225	0.112	0.112

Step 5-6: We apply Eqs. (34-35) to aggregate the q-ROFNs of all attribute to compute the performance value (θ=2) of platform, and we use Eqs.(36-37) to calculate the closeness degree of each platform, then these closeness degrees are shown in Table 8.

Step 7: We use the Eq. (38) to compute the compromise value of each platform, where the compromise coefficient ρ=0.5. The CBG platforms are ranked according to the results, namely, s1>s4>s5>s2>s3. Therefore, the best investment CGB platform is s1. See Table 8.

Table 8
The results and ranking of CGB platforms

Alternatives	$\wp(1)$	$\wp(2)$	$\partial(1)$	$\partial(2)$	Qi	Ranking
NIS	(0.538,0.777)	(0.478,0.811)	-	-	-	-
PIS	(0.782,0.558)	(0.729,0.604)	-	-	-	-
s1	(0.766,0.585)	(0.672,0.688)	0.908	0.667	1.000	1
s2	(0.669,0.655)	(0.564,0.736)	0.510	0.397	0.332	4
s3	(0.622,0.716)	(0.540,0.766)	0.299	0.272	0.000	5
s4	(0.692,0.633)	(0.646,0.676)	0.595	0.658	0.730	2
s5	(0.705,0.627)	(0.640,0.688)	0.634	0.627	0.723	3

5.2 Sensitivity analysis

For exploring the influence of parameters on the decision-making process, the sensitivity analysis of parameters ρ, κ and θ is carried out in this sub-section. We first can get the change of alternatives sorting with changing the value of parameter ρ between the range of [0, 1], as shown in Figure. 2.

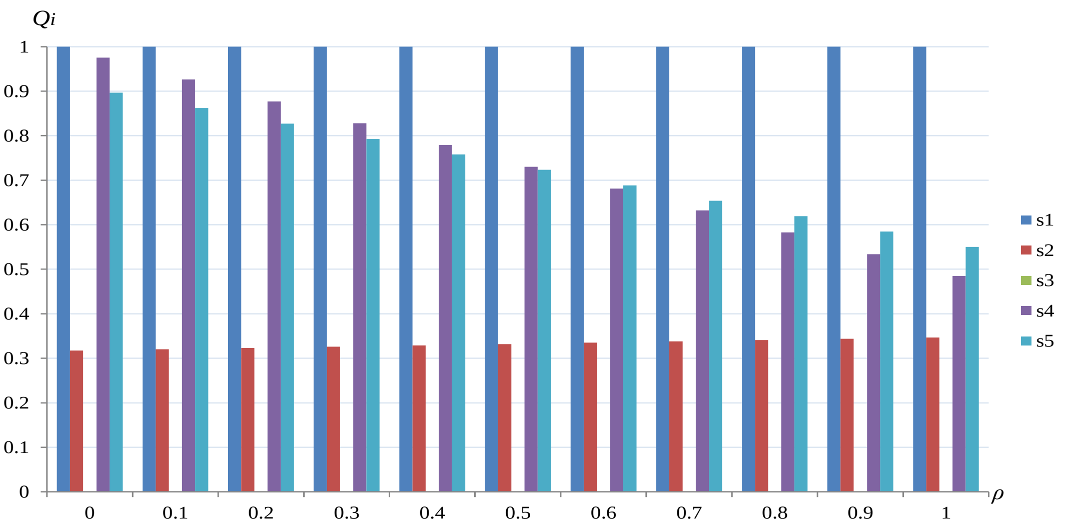


Fig. 2. The performance values of platforms with different parameter ρ

From Figure 2, when the parameter ρ is different in the range of [0.0, 0.5], the platforms ranking is s1>s4>s5>s2>s3, and when the parameter ρ is different in the range of [0.6, 1.0], the platforms ranking is s1>s5>s4>s2>s3. Although the ordering of platforms s4 and s5 varies with different parameter values, the ordering of the optimal platform s1, the worst option s3 and the second worst option s2 remain the same. This means that platforms ranking is stable in the range of parameter $\rho \in [0.0,1.0]$.

Then, we change the parameters κ and θ respectively to obtain the platforms ranking results, which are listed in Tables 9 and 10.

From Table 9, when κ=1 and 2, the ranking of these CGB platforms is s1>s4>s5>s2>s3, and the best option is s1. When κ=3, the order of s1 and s4 is changed, and the order of platforms becomes s4>s1>s5>s2>s3, that is, s4 is the best platform. However, when the value of parameter κ is in the range of [4, 9], the ranking for platforms is s4>s5>s>s2>s3 and remains unchanged. In addition, the optimal platform changes from s1 to s4 as the parameter κ increases from 1 to 3. This indicates that the degree of priority relationship between decision variables can affect the platforms ranking, and it also means that experts determine the parameter κ value according to the actual situation of decision-making scenarios, which to some extent indicates the flexibility of decision-making.

Table 9
The ranking of platforms with different κ (ϑ=2, ρ=0.5)

Parameter	Q1	Q2	Q3	Q4	Q5	Ranking
κ=1	1.000	0.332	0.000	0.730	0.723	s1>s4>s5>s2>s3
κ=2	0.805	0.360	0.000	0.707	0.695	s1>s4>s5>s2>s3
κ=3	0.692	0.358	0.000	0.700	0.689	s4>s1>s5>s2>s3
κ=4	0.626	0.357	0.000	0.699	0.691	s4>s5>s1>s2>s3
κ=5	0.584	0.355	0.000	0.700	0.692	s4>s5>s1>s2>s3
κ=6	0.554	0.354	0.000	0.700	0.693	s4>s5>s1>s2>s3
κ=7	0.532	0.353	0.000	0.700	0.694	s4>s5>s1>s2>s3
κ=8	0.515	0.352	0.000	0.700	0.695	s4>s5>s1>s2>s3
κ=9	0.502	0.351	0.000	0.701	0.696	s4>s5>s1>s2>s3

Table 10
The ranking of platforms with different ϑ (κ=1, ρ=0.5)

Parameter	Q1	Q2	Q3	Q4	Q5	Ranking
θ=2	1.000	0.332	0.000	0.730	0.723	s1>s4>s5>s2>s3
θ=3	1.000	0.330	0.000	0.714	0.712	s1>s4>s5>s2>s3
θ=5	1.000	0.329	0.000	0.698	0.702	s1>s5>s4>s2>s3
θ=10	1.000	0.328	0.000	0.683	0.693	s1>s5>s4>s2>s3
θ=20	1.000	0.327	0.000	0.670	0.684	s1>s5>s4>s2>s3
θ=50	1.000	0.325	0.000	0.656	0.676	s1>s5>s4>s2>s3
θ=100	1.000	0.324	0.000	0.651	0.673	s1>s5>s4>s2>s3
θ=500	1.000	0.319	0.000	0.631	0.658	s1>s5>s4>s2>s3
θ=1000	1000	0.317	0.000	0.638	0.671	s1>s5>s4>s2>s3

From Table 10, when the parameters θ=2 and 3, the ranking of these platforms is s1>s4>s5>s2>s3, while when the parameter θ is larger, the order of platforms remains s1>s5>s4>s2>s3. In the process of parameter θ changing, the optimal platform is still s1, but the ordering of platforms s4 and s5 changes slightly. Similar to the parameterρ, the variation of parameter θ in platform ranking is stable and reliable.

5.3 Comparative study

To exam the effectiveness of the framework we built under q-ROF environment, the proposed AOs are first compared with some existing AOs, and then the improved WASPAS method is compared with the existing ranking technologies in this sub-section. There are some existing AOs, including the q-ROFWA [44], q-ROF weighted geometric (q-ROFWG) [44], q-ROF prioritized weighted average (q-ROFPWA) [42], q-ROF prioritized weighted geometric (q-ROFPWG) [42], q-ROF Dombi prioritized weighted average (q-ROFDPWA) and q-ROF Dombi prioritized weighted geometric (q-ROFDPWG) [43] operators. And there are several existing ranking technologies in q-ROFS environment, including the WASPAS [29], MULTIMOORA (Multiplicative multi-objective optimization by ratio analysis) [43], MABAC (Multi-attributive border approximation area comparison) [49] and EDAS (Evaluation based on distance from average solution) [50]. We apply these decision-making methodologies to obtain the platform ranking results in this case, and the results are shown in Tables 11 and 12, respectively.

Table 11
Ranking of platforms by AOs

AOs	Sc(si)	Ranking
q-ROFWA [43]	Sc(s1)=0.566, Sc(s2)=0.513, Sc(s3)=0.456, Sc(s4)=0.500, Sc(s5)=0.514	s1>s5>s2>s4>s3
q-ROFWG[43]	Sc(s1)=0.364, Sc(s2)=0.374, Sc(s3)=0.367, Sc(s4)=0.426, Sc(s5)=0.416	s4>s5>s2>s3>s1
q-ROFPWA[41]	Sc(s1)=0.700, Sc(s2)=0.438, Sc(s3)=0.312, Sc(s4)=0.610, Sc(s5)=0.623	s1>s5>s4>s2>s3
q-ROFPWG[41]	Sc(s1)=0.643, Sc(s2)=0.309, Sc(s3)=0.257, Sc(s4)=0.565, Sc(s5)=0.569	s1>s5>s4>s2>s3
q-ROFDPWA[42]	Sc(s1)=0.500, Sc(s2)=0.317, Sc(s3)=0.179, Sc(s4)=0.393, Sc(s5)=0.397	s1>s5>s4>s2>s3
q-ROFDPWG[42]	Sc(s1)=0.707, Sc(s2)=0.553, Sc(s3)=0.350, Sc(s4)=0.627, Sc(s5)=0.622	s1>s4>s5>s2>s3
q-ROFFSWA	Sc(s1)=0.625, Sc(s2)=0.509, Sc(s3)=0.437, Sc(s4)=0.539, Sc(s5)=0.552	s1>s5>s4>s2>s3
q-ROFFSWG	Sc(s1)=0.489, Sc(s2)=0.391, Sc(s3)=0.354, Sc(s4)=0.480, Sc(s5)=0.468	s1>s4>s5>s2>s3

From Table 11, the priority level of input arguments is not considered in the q-ROFWA and q-ROFWG operators, then completely inconsistent platform ranking results are obtained from these two AOs, which may lead to decision-making difference. The q-ROFPWA, q-ROFPWG, q-ROFDPWA and q-ROFDPWG operators and AOs proposed all consider the priority relationship of input arguments and the results of information aggregation are basically the same, namely s1 is the best and s3 is the worst. However, the AOLs in the q-ROFPWA and q-ROFPWG operators are simple and have no parameters, so these AOs cannot express DMs' risk preference. Although the q-ROFDPWA and q-ROFDPWG operators contain Dombi operators, when the MD μ or ND ν is zero in q-ROFNs, these two AOs cannot be adopted, because the denominator of Eqs. (39-40) cannot be zero. In contrast, the proposed AOs not only consider the priority relationship of input arguments but also generalize the priority. Meanwhile, the monotonicity of the score functions of these two AOs with respect to parameter θ can reflect the risk preference and decision attitude of DMs according to FOLs in q-ROFNs. Therefore, compared with existing AOs, the proposed AOs are more reasonable and comprehensive, and more suitable for aggregating evaluation information in actual decision-making problems.

$$q-ROFDPWA(P_1,P_2,\cdots,P_\sigma)=\left(\sqrt[q]{1-\frac{1}{1+\left\{\sum_{\xi=1}^{\sigma}\left(\frac{\lambda_\xi\chi_\xi}{\sum_{\xi=1}^{t}\chi_\xi}\right)\left(\frac{\mu_\xi^q}{1-\mu_\xi^q}\right)^\eta\right\}^{1/\eta}}},\sqrt[q]{\frac{1}{1+\left\{\sum_{\xi=1}^{\sigma}\left(\frac{\lambda_\xi\chi_\xi}{\sum_{\xi=1}^{t}\chi_\xi}\right)\left(\frac{1-\nu_\xi^q}{\nu_\xi^q}\right)^\eta\right\}^{1/\eta}}}\right) \quad (39)$$

$$q-ROFDPWG(P_1,P_2,\cdots,P_\sigma)=\left(\sqrt[q]{\frac{1}{1+\left\{\sum_{\xi=1}^{\sigma}\left(\frac{\lambda_\xi\chi_\xi}{\sum_{\xi=1}^{t}\chi_\xi}\right)\left(\frac{1-\mu_\xi^q}{\mu_\xi^q}\right)^\eta\right\}^{1/\eta}}},\sqrt[q]{1-\frac{1}{1+\left\{\sum_{\xi=1}^{\sigma}\left(\frac{\lambda_\xi\chi_\xi}{\sum_{\xi=1}^{t}\chi_\xi}\right)\left(\frac{\nu_\xi^q}{1-\nu_\xi^q}\right)^\eta\right\}^{1/\eta}}}\right) \quad (40)$$

where η is operational parameter and η>0. Pξ=(μξ,νξ) (ξ=1, 2,..., σ) is a set of q-ROFNs, and λξ is the corresponding weight value and $\sum_{\xi=1}^{\sigma}\lambda_\xi=1$,λξ > 0. $\chi_\xi=\prod_{i=1}^{\xi-1}Sc(P_i)$ and χ1=1. Sc(Pξ) is the value of score function.

Table 12
The results of different methods with *q*-ROFNs

Methods	Performance values of platforms	Ranking
MABAC[49]	U_1=0.160, U_2=0.077, U_3=-0.085, U_4=0.028, U_5=0.049	s_1>s_2>s_5>s_4>s_3
EDAS[50]	U_1=0.770, U_2=0.512, U_3=0.120, U_4=0.438, U_5=0.452	s_1>s_2>s_5>s_4>s_3
WASPAS[29]	U_1=0.479, U_2=0.448, U_3=0.411, U_4=0.466, U_5=0.471	s_1>s_5>s_4>s_2>s_3
MULTIMOORA[43]	RS: U_1=0.500, U_2=0.317, U_3=0.179, U_4=0.393, U_5=0.397 RP: U_1=0.942, U_2=0.909, U_3=0.924, U_4=1.000, U_5=0.908 MF:U_1=0.707, pU_2=0.553, U_3=0.350, U_4=0.627, U_5=0.622	s_1>s_4>s_5>s_2>s_3
This paper	Q_1=1.000, Q_2=0.332, Q_3=0.000, Q_4=0.730, Q_5=0.723	s_1>s_5>s_4>s_2>s_3

In Table 12, the MABAC, EDAS, MULTIMOORA, WASPAS methods and the improved WASPAS method have all obtained the best platform s1 and the worst platform s3 when applied to this case, while the ranking of other platforms is different. The WASPAS [29] and our improved WASPAS methods get the same ranking, as shown in Table 12 and Figure 3.

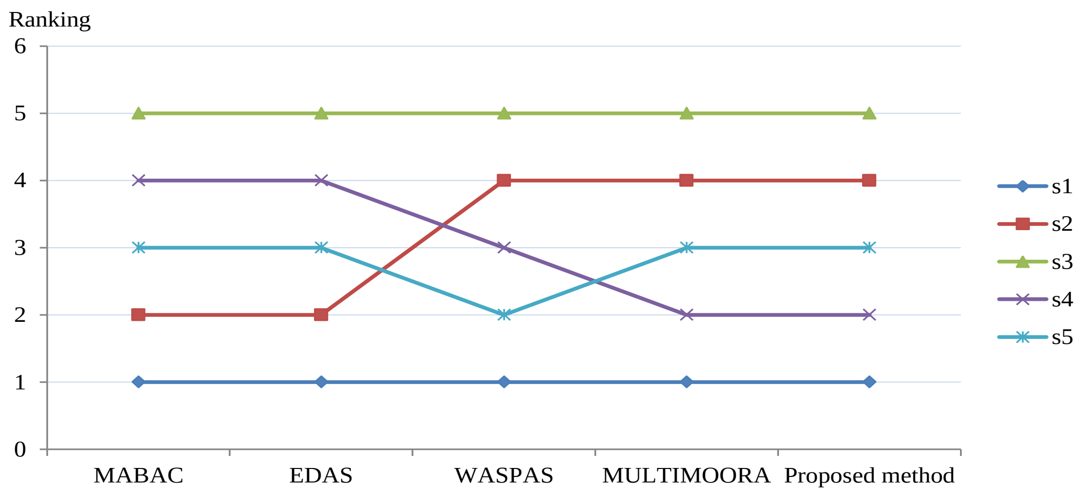


Fig. 3. Ranking of platforms by different decision methods

Specifically, there are differences between the above decision methods. The existing methods employ the q-ROFWA or q-ROFWG operator to fuse individual evaluation information of experts, while this paper utilizes the q-ROFFSWA and q-ROFFSWG operators to aggregate individual assessment information respectively. In this way, the true reflection of the priority level of experts and the consistency and comprehensiveness of their consideration of risk preference can be ensured, while the q-ROFWA or q-ROFWG operator can neither capture the priority relationship of experts nor reflect their risk preference and decision-making attitude. In terms of de fuzzy technology, this paper abandon the score function method for de fuzzy the final value of the alternative in WASPAS method [29], and adopted the distance measure technology instead. In this way, partial information loss caused by the shortcomings of score function in comparing sizes of different q-ROFNs can be avoided. In terms of decision mechanism, the proposed AOs contain the parameter θ and the monotonicity on θ, which are sufficient to reflect the two conflicting risk preferences of experts, and can continue the risk preferences to the final value through the parallel calculation process. Therefore, the parameter ρ in the improved WASPAS method endows the compromise meaning between two different risk preferences and decision attitudes. This is also the most essential difference between this article and literature [29].

To emphasize the superiorities of the improved WASPAS, the characteristics of different existing methods are compared and analyzed. See Table 13.

Table 13
Comparison of features with existing methods under *q*-ROFS environment

Features	Wang *et al.* [49]	Li *et al.* [50]	Aydemir & Gündüz [43]	Rani and Mishra [29]	This paper
Decision type	Group	Group	Single	Group	Group
Methods	MABAC	EDAS	MULTIMOORA	WASPAS	Modified WAPAS
AOs for individual evaluation information aggregation	q-ROFWA OR q-ROFWG	q-ROFWA OR q-ROFWG	-	q-ROFWA OR q-ROFWG	q-ROFFSWA AND q-ROFFSWG
AOs for utility values of alternatives	q-ROFWG	q-ROFWA	q-ROFDPWA and q-ROFDPWG	q-ROFWA and q-ROFWG	q-ROFFSWA and q-ROFFSWG
Defuzzification technologies	Distance measure	Score function	Distance measure and Score function	Score function	Distance measure
Decision-making mechanism	Border approximation area comparison	Comparison of distance from average solution	Dominance theory	Linear weighted sum method	Compromise the opposite risk performance
Decision flexibility level	NO	NO	Medium level	Weak level	Strong level
Whether consider the priority of variables	NO	NO	YES	NO	YES
Whether consider the generalization of variable priority level	NO	NO	NO	NO	YES
Whether consider the risk preference and decision attitude of experts	NO	NO	NO	NO	YES

6 Conclusion

The WASPAS method is a simple and practical ranking technology, which is often used to solve various real decision-making problems. The traditional WASPAS method relies on WSM and WPM models, de fuzzies by score function, and obtains the final utility of the alternative by linear weighted sum model. However, this method not only fails to reflect DMs' risk preference and decision attitude, but also ignores the priority level of decision variables. To remedy these defects, we introduce the FOLs and SoftMax function in q-ROF environment, and propose two AOs, including the q-ROFFSWA and q-ROFFSWG operators. We not only discuss these AOs' properties and special cases, but also analyze the monotonicity of these AOs' score functions on the parameter θ, which can express the opposite decision attitudes with risk preference of DMs. Furthermore, we construct a new WASPAS framework based on these AOs and distance measures to settle the q-ROF MAGDM problems. We utilize the q-ROFFSWA and q-ROFFSWG operators respectively to aggregate decision variables (including individual expert evaluation information and performance values of alternatives with different attributes), and de fuzzy them by distance measure from PIS and NIS. Then, a new compromise function which can balance the optimistic and pessimistic decision attitudes with risk preference is constructed to calculate the final compromise value of the alternative. Finally, we apply this framework to a real case of investment decision on CGB platform, and verify the effectiveness and practicability of the improved WASPAS by performing sensitivity analysis and comparative study.

There are some advantages, which are summarized as follows:

(1) The proposed AOs contains parameters θ and κ, which can not only reflect the decision attitude with risk preference of decision-makings, but also capture the priority relationship among decision variables and generalized priority level, so as to improve the flexibility of decision making process.

(2) The q-ROFWA or q-ROFWG operator is commonly utilized to fuse individual evaluation information of experts in existing q-ROF group decision problems. Different from this, the q-ROFFSWA and q-ROFFSWG operators are independently applied in this paper to calculate the evaluation information from individual experts in parallel and aggregate the performance values of alternatives with different attributes, to accurately characterize the decision attitude with risk preference of experts.

(3) On the basis of the extended group q-ROFDM, we calculate the Hamming distance between the PIS and NIS to de fuzzy the performance value of the alternative, which can overcome the defect of the existing score function to compare the size of q-ROFNs, so as to avoid partial information loss.

(4) We construct a compromise function in the WASPAS method. Compared with the linear weighted sum method in literature [29], the parameter ρ is endowed with theoretical and practical significance to balance two completely opposite decision attitudes with risk preference of DMs.

Therefore, we will further extend the proposed method to circular intuitionistic fuzzy sets [51], T-spherical fuzzy sets [52], Complex q-ROFSs [53], Complex T-spherical fuzzy sets [54] and other decision-making environments, and to deal with practical decision-making problems, such as risk assessment, site selection and supplier evaluation.

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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