

Facility Location Optimization For Technical Inspection Centers Using Multi-Objective Mathematical Modeling Considering Uncertainty

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ABSTRACT

Article history:	Encountering numerous vehicles on the roads can pose several risks,
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Available online 9 September 2023	easily accessible. To provide the most customer service coverage at the
<i>Keywords:</i> Facility Location; Multi- Objective Optimization; Uncertainty; Non- dominated Sorting (NSGA-II).	lowest cost of transportation for technical inspection centers, facility location optimization is proposed in this paper. Specifically, we investigate the location of technical inspection centers (TICs) as a maximum coverage problem while minimizing the cost of TIC locations' construction and customers' transportation. To deal with this problem, we propose a robust programming considering our numeric data's uncertainty. Our research contributes to facility location optimization by providing a novel insight into solving the problem using a hybrid mathematical model. It presents a two- objective linear optimization model with binary variables to address this optimization problem. We used the Augmented Epsilon Constraint (AEC) method via the CPLEX solver and the Non-dominated Sorting Genetic Algorithm II (NSGA-II) method for large-scale problems to solve the model. A case study was conducted to test the numerical analysis methodology and several practical problems of varying scales. The final results demonstrate the effectiveness of the proposed approach in meeting the optimality and feasibility robustness criteria. Identifying optimal TIC locations regarding the paper's main objective proves the advantage of using the mentioned innovative methodology.

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1. Introduction

The role of vehicle defects in causing road accidents has been acknowledged and accepted. Reports have provided road crashes attributed to vehicle defects, ranging from 3% to 19% in developed nations [1-2]. Simultaneously, advancements in the automotive industry have led to an increased emphasis on technical inspections to enhance safety. One effective way of achieving this is by establishing check stations that provide inspections. Yan *et al.*, [3] and Rababah *et al.*, [4] highlighted the importance of standardized testing procedures to ensure accurate and reliable inspection results. Similarly, Kim *et al.* [5] examined the effectiveness of inspection methods for identifying damage in vehicles.

An optimization approach is required to determine the optimal locations for TICs which is critical to ensure their minimum time and cost of accessibility to drivers. In this regard, Balinski [6] introduced the facility location problem (FLP), which deals with determining the optimal location of a new group of facilities such that the combined costs are minimized. Noura *et al.*, [7] emphasized the importance of considering accessibility, coverage, and public satisfaction when deciding where to locate inspection centers. However, number of applicants for technical inspection is dispersed across multiple regions, and demand rate can be uncertain. Therefore, an optimization approach must be taken under uncertain conditions, focusing on controlling uncertainty in parameters to establish technical inspection centers considering desired objectives.

High-level uncertainty is an unavoidable characteristic in many operations [8-9]. Despite the lack of precise statistics on the number of vehicles in Iran, researchers have developed various methods for estimating this number, such as using per capita vehicle ownership rates and population data [10].

In this research, the potential population in each region and the estimated demand rate for technical inspection are still being determined. This makes the problem more complex, and the solution must be determined to reduce decision-making risk. It is assumed that the uncertain data is expressed as fuzzy approximated numbers. These approximations are expressed as triangular or trapezoidal fuzzy numbers that reduce the risk of decision-making based on nominal data [11]. Fuzzy set theory offers a powerful tool to deal with various types of uncertainties, including fuzzy coefficients [12]. Thus, the theory provides a comprehensive framework for simultaneously handling different kinds of uncertainties [13]. In this research, the Robust Possibilistic Programming (RPP) approach has been used to deal with uncertainty. Then, the evolved epsilon-constraint method was employed to balance objectives.

Finding the optimal location is not a one objective problem to be solved, A paper by Lan *et al.* [14] asserts that a significant portion of location-related issues have been examined under ideal conditions. However, the economic landscape is becoming increasingly complex in practice.

Equally important, it is necessary to take into account multiple other factors when optimizing our model, as we are simulating a real-life case [15]. Therefore, the problem becomes increasingly complex and requires innovative approaches to be effectively addressed [16]. The objective of facility location is to identify suitable options, considering constraints and objectives such as cost reduction. Facility location decisions are typically long-term and involve addressing various conflicting objectives, including customers and distances [17-18]. As a result,

finding optimal solutions to facility location requires the development of novel approaches. In numerous practical scenarios, it is common to seek multiple objectives to make the best use of the resources available [19-20]. This approach can turn the problem into a Multi-Objective Problem, where the objectives may occasionally contradict each other [21].

For example, Wang *et al.*, [22] utilized a genetic algorithm and Geographic Information Systems (GIS) in a multi-objective optimization model to determine the location for vehicle inspection stations. Similarly, Wang *et al.*, [23] developed a hybrid algorithm and GIS-based location model for electric vehicle (EV) charging stations, considering criteria such as EV battery capacity and station distance. Additionally, Cho *et al.*, [24] proposed a method that combined GIS and multi-criteria decision-making to optimize the locations of inspection facilities for heavy vehicles.

Facility location problem has NP-hard nature. Researchers often opt heuristic algorithms to tackle this problem due to their ability to handle complex scenarios. Techniques such as neural networks, fuzzy logic, and evolutionary computations can be leveraged to overcome these complexities [25]. Boloori *et al.*, [26] have categorized meta-heuristic algorithms into two main types: single and multi-objective models which NSGA-II is a well-known and widely used multi-objective optimization algorithm with unique features among them [27]. For Instance, Gao [28] developed two uncertain models to address single facility location problems on a network, while Wen *et al.*, [29] proposed an uncertain facility location-allocation model using chance constraints. Wang *et al.*, [30] investigated two uncertain programming models for hierarchical facility location problems in uncertain environments. Wu *et al.*, [31] presented an uncertain chance-constrained model to address logistics distribution center location problems under uncertainty. The studies mentioned above, along with others, serve as evidence of the efficacy of meta-heuristic techniques in resolving complex problems.

The novelty of this research's applied model lies in its unique combination and its consideration of uncertainty. Two optimization models are presented to address the problem. The first model is a two-objective mathematical optimization model that assumes all parameters/data are certain. In contrast, the second model is a robust optimization model that accounts for uncertainty. A precise two-objective solution method is provided for small and medium dimensions to accommodate varying problem sizes [32]. Additionally, for larger-scale problems, a two-objective metaheuristic approach based on Genetic Algorithms and Non-dominated Sorting (NSGA-II) is utilized. The ultimate goal of this mathematical model is to determine optimal locations for technical inspection centers in Tehran, Iran.

2. Literature Review

2.1. Location-allocation models

The problem of finding an optimal location has been discussed by ReVelle et al., [33]. Location-allocation models are optimization models created to identify the most optimal location for providing services within a city or region, taking into account the geographical dispersion of demand for said service.

2.1.1. Maximal covering location problem (MCLP)

The initial models in this category were referred to as LSCP or location set covering problems [34]. However, they produced costly solutions because they didn't account for the cost of covering remote areas. To overcome this limitation, the maximal covering location problem, MCLP, was introduced.

Cappanera *et al.*, [35] have discussed applications of coverage problems in both private and government sectors. Aboolian *et al.*, [36] and Yavary *et al.*, [37] have examined a generalized version of the covering location problem, which allows for partial coverage of customers. Arabani *et al.*, [26] have comprehensively examined and studied coverage location problems, focusing more on research articles published after the Schilling *et al.*, [38] paper. The dynamic maximum coverage location problem (DMCLP) has been identified and examined as a research gap. Subsequently, this problem has been solved using a simulated annealing algorithm [39].

2.2. Uncertainty

The gap between reality and the unknown is referred to as uncertainty, which means that half of the data and information exist incompletely. Ho [40] classified uncertainty into two categories: system uncertainty and dynamic uncertainty. Liu [41] presented the theory of set uncertainty as a generalization of uncertainty theory to the domain of uncertain sets.

2.2.1. Dealing with uncertainty

The main approaches that are commonly used in dealing with uncertainty in the field of mathematical programming are as follows: Stochastic, Fuzzy, Robust. A robust Optimization is a risk-averse approach to dealing with uncertainty in optimization problems. According to the scientific definition provided by Pishvaee *et al.*, [42], a robust solution must fulfill two conditions: 1. It should remain feasible for all uncertain parameters.

2. It should be close to its optimal value for all uncertain parameters.

2.3. Multi-Objective Optimization

Most optimization problems are multi-objective, meaning that in their solution, various performance criteria, which are usually contradictory, must be considered simultaneously. As a result, existence of conflicting objectives makes it impossible to achieve a single optimal solution that satisfies all objectives [43]. Two multi-objective optimization (MOO) methods do not require complex mathematical equations: Pareto and scaling.

2.4. Previous studies

In Table 1, related research and contributions will be reviewed and the existing gap will be discussed.

Table 1

Summary of Conducted Research

#	Author	Year	Title	Explanations		Uncertainty
1	Daskin <i>et al.,</i> [44]	2005	Facility Location in Supply Chain Design	Bi-objective optir	nization	Yes
2	Harris <i>et al.,</i> [45]	2009	The multi-objective uncapacitated facility location problem for green logistics	single-objective optimization		No
3	Gülpınar <i>et</i> <i>al.,</i> [46]	2013	Robust strategies for facility location under uncertainty	Random facility location.	A strong optimal strategy in the worst-case scenario.	Yes
4	Berrocal- Plaza <i>et al.,</i> [47]	2014	On the use of multi-objective optimization for solving the Location Areas strategy with different paging procedures in a realistic mobile network	Location Allocation	Multi- objective Optimization	No
5	García Quiles <i>et al.,</i> [48]	2015	Covering Location Problems	Facility location	Maximum coverage	No
6	Zhang <i>et al.,</i> [49]	2016	A multi-objective optimization approach for health-care facility location-allocation problems in highly developed cities such as Hong Kong	Multi-objective optimization	Location Allocation	No
7	Karatas <i>et</i> <i>al.,</i> [50]	2018	An iterative solution approach to a multi-objective facility location problem	Multi-objective facility location	Coverage	No
8	Wang <i>et al.,</i> [51]	2018	Multi-objective competitive location problem with distance-based attractiveness for two facilities	Multi-objective facility location		No
9	Lee <i>et al.,</i> [52]	2019	Multi-objective optimisation of hybrid power systems under uncertainties	Multi-objective fa	acility location	Yes
10	Li <i>et al.,</i> [53]	2020	Wind Integrated Power Systems by Multi-objective Optimization Approach	Multi-objective Optimization		Yes
11	Qi <i>et al.,</i> [54]	2022	A self-exploratory competitive swarm optimization algorithm for large-scale multi-objective optimization	large-scale multi-objective optimization		No
12	Liu <i>et al.,</i> [55]	2023	Property of decision variables- inspired location strategy for multi-objective optimization	Location Allocation	Multi-objective optimization	No
13	Wang <i>et al.,</i> [56]	2023	Differential evolution guided by approximated Pareto set for multi-objective optimization	Pareto Set	Multi-objective optimization	No

#	Author	Year	Title	Explanations		Uncertainty
14	Gu <i>et al.,</i> [57]	2023	A chaotic differential evolution and symmetric direction sampling for large- scale multi-objective optimization	large-scale multi- optimization	objective	No
15	Current Resea	irch		Multi-objective optimization	Maximum Coverage	Yes

2.4.1. Research Gap

Previous research has explored the areas of facility location, allocation, and uncertainty, and researchers have identified research gaps within these areas. This study's objective is to maximize the coverage of demand areas for technical inspections while minimizing the costs of construction and service provision. It also incorporates uncertainty in the population of demand and the demand rate in each region.

3. Methodology

3.1 Problem Definition

This research aims to determine the optimal location of technical inspection centers within a specific territory in a way that achieves two objectives: "maximizing the coverage of demand areas" and "minimizing the distance/cost of transportation between demand areas and inspection centers." Three capacity levels, small, medium, and large, are considered for establishing these centers.

Several potential locations have also been considered for establishing new centers. The population in each area can visit one or multiple centers, and each center can serve one or multiple areas (subject to the limited capacity of each inspection center). In the classic maximum coverage problem, facility location is determined to maximize the coverage of demand areas to the facilities while considering the constraint of the maximum number of facilities to be established. The objective of this research is also to achieve these goals in locating the technical inspection centers. However, it also considers minimizing the cost of establishing the inspection centers and the transportation costs between the demand areas and the established inspection centers considering constraints such as:

- The limited capacity of inspection centers
- Maximum number of established inspection centers
- Maximum queue length at each inspection center

Notably, this research problem is formulated as a location-allocation problem of maximal coverage type, as economic objectives are also considered, making it a multi-objective problem that requires balancing between the mentioned objectives. Moreover, considering the uncertainty of problem parameters, an approach must be employed to handle the uncertainty effectively. The main assumptions of the research problem are as follows:

- 1. Potential locations for establishing technical inspection centers are predetermined.
- 2. A limited coverage radius is defined, and each inspection center can only cover areas within this radius.

- 3. The cost of establishing inspection centers at different locations with different capacities is variable.
- 4. The maximum number of inspection centers to be established is predetermined.
- 5. The distance between any two points in the network is predetermined.
- 6. All demand regions must not be assigned to a single inspection center. A single inspection center can serve multiple regions.
- 7. Only light vehicles are considered for inspection.
- 8. Service rate, maximum queue length, and minimum queue probability are assumed to be identical for all inspection centers.
- 9. It is optional to cover all demand or applicant regions.
- 10. If the potential population for inspection in a demand region is less than 10,000, then that region must be connected to an inspection center.
- 11. The potential population in each demand region and demand rate are uncertain and estimated as fuzzy numbers.

3.2. Modeling Methodology

3.2.1. Mathematical Symbols and Notations

The mathematical symbols and notations used in modeling and solving the problem are as follows: Index Set

$I = \{1, 2,, i,, I = N\}$	Potential locations for the construction of centers
$K = \{1, 2, \dots, k, \dots, K = 3\}$	Capacity levels for centers, 1- low, 2- medium, 3- high.
$J = \{1, 2,, j,, J = N\}$	Services/Tasks
Parameters/Data $\widetilde{lpha_j}$	The potential population in region j applying for services (estimated fuzzy number)
$\widetilde{z_j}$ R	The demand rate of region j for services (estimated fuzzy number) radius of coverage
S	maximum number of centers
cap_k	the capacity of the center with size k
f_{ki}	The construction cost of the center in region i with size k
d_{ij}	distance of center i from region j
С	the unit cost of transportation
μ	Service rate in each center
β	the maximum length of the queue at each center.
π	the minimum probability that the queue length does not exceed β .

Variables/Outputs

 y_{ik}

1: if a center with k capacity is built in the location i

0: otherwise

x_{ij}

Share of the total demand of region j that is dedicated to the center i. (a continuous variable between 0 and 1)

$$\operatorname{Max} F_{1} = \sum_{i} \sum_{j} \widetilde{a}_{j} x_{ij}$$
(1)

$$\operatorname{Min} F_{2} = \sum_{i} \sum_{k} f_{ik} y_{ik} + \sum_{i} \sum_{j} c. d_{ij}. \widetilde{a_{j}}. x_{ij}$$

$$\tag{2}$$

Eq. (1). represents the primary objective function of the problem, which maximizes the coverage of demand for technical inspections. Eq. (2). minimizes the total cost, which includes the cost of establishing centers and the cost of transporting from demand areas to centers. To have same objectives, we rewrite the first objective function as minimizing the population covered. Let $W = \sum_j \tilde{a_j}$ be the total potential population for inspection. Then, the objective function is equivalent to Min $W - \sum_i \sum_j \tilde{a_j} x_{ij} = (\sum_i \sum_j \tilde{a_j} (1 - x_{ij}))$. We denote this value with the symbol TWU. Additionally, since F_2 directly calculates the cost, we use the symbol **Cost** for it. The constraints are as follows:

$$x_{ij} \le \sum_{k} y_{ik} \quad ; \forall i,j \tag{3}$$

• Eq. (3). guarantees that allocating areas requesting examination to centers requires the center to be built with a specific capacity.

$$\sum_{j} \widetilde{z}_{j} \cdot \widetilde{a}_{j} \cdot x_{ij} \leq \sum_{k} cap_{k} y_{ik} \quad ; \forall i$$
(4)

• Eq. (4). guarantees the limited capacity of each center.

$$\sum_{k} y_{ik} \le 1 \quad \forall i \tag{5}$$

• Based on Eq. (5)., a maximum of one center of a specific size can be placed in each potential location for establishing centers.

$$\sum_{i} \sum_{k} y_{ik} \le S \tag{6}$$

• Eq. (6). shows the maximum number of centers that can be built from N potential locations.

$$u_{ij} = \begin{cases} 1; \frac{R}{d_{ij}} \ge 1 \ \forall i, j \\\\ 0; \frac{R}{d_{ij}} < 1 \ \forall i, j \end{cases}$$

• In Eq. (7)., an auxiliary binary variable u_{ij} is defined and it is 1 as long as the distance of potential center i is less than area j. $\left(\frac{R}{d_{ij}} < 1 \rightarrow u_{ij} = 0\right)$; $\frac{R}{d_{ij}} \ge 1 \rightarrow u_{ij} = 1$).

$$\sum_{i} u_{ij} > \frac{\tilde{a_j}}{10000} \quad \forall j$$
(8)

• Eq. (8). ensures that if the population of an area is more than 10,000, then it must be within the coverage radius of at least one center. ($\exists i : u_{ij} \ge 1$)

$$\sum_{j} \widetilde{z}_{j} \cdot x_{ij} \le \mu \cdot \sqrt[\beta+2]{1-\pi}$$
(9)

- Finally, the variables of the problem and the range of their variations are specified in the Eq. (10).

3.2.2. The solution method of the proposed two-objective model

The suggested approach to solve the two-objective model of facility location is based on a mixed-integer linear programming (MILP) model. By employing an accurate method to solve linear two-objective problems and utilizing the CPLEX Solver, it is possible to obtain the globally optimal solution for the problem. The general form of an MODM problem is as follows:

$$\begin{cases} Min\left(f_1(x), f_2(x), \dots, f_n(x)\right) \\ x \in X \end{cases}$$
(11)

It is assumed that the first objective is considered the primary objective. The other objectives are constrained within an upper bound of the limited epsilon values and incorporated as constraints in the problem. In this case, the EC method is employed, and the following single-objective model is obtained:

$$\begin{cases} Min \ f_1(x) \\ f_i(x) \le e_i \ i = 2, 3, \dots, n \\ x \in X \end{cases}$$
(12)

Modifying/completing the model can address the issue of Weakly Efficiency, which is called the AEC method. For better implementation of the Epsilon constraint (EC) method, one can first obtain the appropriate range of epsilon values (e_i) using the Lexicographic method. In the AEC method, it is necessary to determine the suitable range of variations for the values and then obtain the Pareto front for different values. The AEC method is as follows:

(7)

$$\begin{cases} Min \ f_1(x) - \sum_{i=2}^n \phi_i s_i \\ f_i(x) + s_i = e_i \ i = 2, 3, \dots, n \\ x \in X \\ s_i \ge 0 \end{cases}$$
(13)

In which, the variables s_i represent non-negative variables for deficiencies, and ϕ_i is a parameter used to normalize the value of the first objective function concerning the i objective. $(\phi_i = \frac{R(f_1)}{R(f_i)}).$

In the proposed AEC method in this study, the $e_i \in [Min(f_i), Max(f_i)]$ the range is initially determined using the Lex approach for the constrained objectives. Subsequently, a single-objective model is solved by assigning values to the e_i variables, resulting in an efficient solution with objective values on the Pareto front. It is important to note that by altering the e_i values within their respective ranges, different efficient solutions, and points on the Pareto front can be obtained. The following section explains the application of the AEC method to solve the proposed two-objective model.

3.2.3. Utilization of the Enhanced Epsilon Constraint Evolutionary (EEC) method for solving the research problem

In this study, to apply the AEC method in solving the defined UPMS_MFC problem, we designate the first objective function ($f_1 = TWU$) as the main objective and restrict the second objective ($f_2 = Cost$)to various limited epsilon values. After determining the payoff matrix ($PayOff = [payOff_{ij}]_{2*2}$) the key steps in the process of employing the AEC method are as follows:

- 1. Calculation of the minimum, maximum, and range of variations for objectives (i=1,2):
 - $Min(f_i) = Min_j \{ payOf f_{ij} \} = payOf f_{ii} \}$
 - $Max(f_i) = payOff_{ij}$
 - $R(f_i) = Max(f_i) Min(f_i)$
- 2. Initialize epsilon for the second objective: $Epsilon = Min(f_2)$
- 3. Partition the range of variations of epsilon into N segments
- 4. Determine the step size for changing epsilon: $StepSize = \frac{R(f_2)}{N-1}$
- 5. Define constraints using the AEC method for the second objective: $f_2 + slack_2 = Epsilon$, Where slack_2 is a non-negative auxiliary variable corresponding to the second objective.
- 6. Define normalization values: $\phi = \frac{R(f_1)}{R(f_2)}$
- 7. Define the objective function using the AEC method: $z = f_1 \phi$. $Slack_2$
- Define a loop to solve an Nth-order optimization problem with the step function 8 and consider the constraint function 6 alongside other constraints for different values of Epsilon.

3.2.4. Solution method for large-scale dimensions

The problem of facility location and assignment falls under the category of highly difficult (NP-hard (Non-Polynomial Deterministic Hard problem)) problems, which means that no polynomial-time algorithm has been discovered to solve this complex problem efficiently. Therefore, employing mathematical programming models (such as MILP used by Gharibi *et al.,* [58]) and using solvers like CPLEX to solve this problem in large-scale dimensions is ineffective and requires significant time and cost. Various metaheuristic algorithms have been proposed to solve the facility location and assignment problem in different topics [59]. Numerous research studies have used Genetic algorithms (GA), Simulated Annealing (SA), Monte-Carlo simulation [60], and some hybrid algorithms.

In this research, metaheuristic algorithms are also employed to solve the problem in largescale dimensions. Here, GA is used, but considering that the problem in question is bi-objective, the bi-objective version of GA called Non-Dominated Sorting Genetic Algorithm II (NSGA-II) is utilized. NSGA-II is based on GA and performs non-dominated sorting of objective values using a sorting algorithm. The details of GA are omitted; however, the general concepts of NSGA-II are explained further, followed by a complete description of its application to the problem addressed in this research.

3.2.5. The proposed algorithm is a genetic-based metaheuristic, and its application for multiobjective optimization modelling

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is one of the versions of genetic algorithms used for solving multi-objective optimization problems [61]. When using a multi-objective algorithm to solve a multi-objective problem, at least two objective functions are considered [62]. It is difficult to definitively determine the superiority of solutions since some points are not utterly superior to others, making pairwise dominance comparisons impossible. This algorithm assigns ranks to solutions based on how many others they dominate. The first front consists of non-dominated points with rank one, while solutions dominated only by the first front are placed in the second front with rank two, and so on. At the end of the algorithm, solutions with rank one, representing the best rank, are selected as the Pareto front points or solution set.

According to Figure 1, first, a group of individuals who have never been defeated is identified and assigned a rank of one. Then, for the remaining members, disregarding the impact of rank one individuals on the population, the sorting of undefeated individuals is done again, and those who have never been defeated in this stage are assigned a rank of two. For the rest of the members, disregarding the impact of rank one and two individuals on the population, the sorting of undefeated individuals on the population, the sorting of undefeated individuals is done again, and those who have never been defeated individuals is done again, and those who have never been defeated individuals is done again, and those who have never been defeated individuals is done again, and those who have never been defeated in this stage are assigned a rank of three. This process continues until the rank of all population members is determined.



Fig. 1. A Sample of Pareto Front

The crucial elements of this algorithm are determining the chromosome structure (solution representation) and the neighborhood structure, which encompasses crossover, mutation, and the evaluation function. The chromosome structure needs to be carefully designed to encompass as many variables of the model as possible while incorporating numerous model constraints. Moreover, this structure should facilitate the straightforward application of crossover and mutation operators. Similarly, the crossover and mutation structures should be defined in a manner that allows for a thorough exploration of the solution space and enables the generation of high-quality solutions using these operators.

4. Solution Methods

This chapter addresses problem-solving, output reporting, and result analysis. To accomplish this, we begin by conducting a numerical study of the research problem proposed in the previous chapter. We then apply the proposed model data to correspond with this study. Subsequently, we solve the problem using each of the suggested solution approaches and provide a report on and analysis of the obtained numerical results. The third section of this chapter focuses on evaluating the proposed heuristic method. We randomly generate several practical problems of various dimensions and solve those using precise and heuristic approaches. Ultimately, we evaluate the performance of each approach based on multiple criteria.

4.1 Numerical Study

The numerical study focuses on the location-allocation problem for technical inspection centers, explicitly examining a network encompassing 22 districts in Tehran. It should be noted that the geographical distances in this network are determined using the resources from Jabarzadeh *et al.* [63]. Tehran, the capital city, is divided into 22 districts, with particular attention given to the service provision in District 1, which consists of 10 sub-regions. Table 2 presents the polar coordinates of these sub-regions.

Table	2
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coordinates of the Exam			
Network Node	Longitude	Latitude	
District 1 Part 1	51.422089	35.810361	
District 1 Part 2	51.433585	35.794213	
District 1 Part 3	51.442770	35.815933	
District 1 Part 4	51.451183	35.797552	
District 1 Part 5	51.461568	35.804727	
District 1 Part 6	51.475557	35.815442	
District 1 Part 7	51.483200	35.809735	
District 1 Part 8	51.483969	35.799782	
District 1 Part 9	51.509809	35.805697	
District 1 Part 10	51.504483	35.791914	
District 2	51.36222	35.7575	
District 3	51.44806	35.75444	
District 4	51.49194	35.74194	

Coordinates of the Examined Regions

In this case study, there are ten sub-regions in District 1, along with Districts 2, 3, and 4, which are adjacent to District 1 and have shorter distances to its sub-regions. These locations are being considered potential/candidate sites for establishing inspection centers. The problem size for this study is presented in Table 3.

Table 3

The Dimensions of the Problem in Numerical Study

I = N	J = M	K
Potential locations	Applicant areas	Capacity levels
13	10	3

Using provided data, the network's distance between two points, A and B, can be calculated based on their geographical latitude and longitude coordinates using the following mathematical relationship:

 $Dis(A,B) = 6371.1 \times \arccos[sin(Lat_A) \times sin(Lat_B) + cos(Lat_A) \times cos(Lat_B) \times cos(Long_A - Long_B)]$, Other parameters are listed in Table 4.

Table 4

Parameters/Data Considered for Numerical Study

Parameter	Value
$\widetilde{a_l}$	Uniform (400,500)
$\widetilde{\widetilde{z_{l}}}$	Uniform (0.1,0.3)
Ŕ	10 km
S	5
cap_k	Uniform (700,1000)
f_{ki}	Uniform (100,300) \$
d_{ij}	Distance based on the given table and formula
c	Uniform (3,5) \$
μ	0.80
β	50
π	0.90

4.2. Solving and Numerical Results

In this section, a mathematical programming model and the precise AEC method are utilized to solve the bi-objective problem in this numerical study, aiming to derive the global Pareto front (Figure 2). Subsequently, the NSGAII method is employed for the same problem, and its Pareto front (Figure 3) is compared with the Pareto front obtained from the combined AEC method in Table 5 and Table 6.

Trade-Off between objectives using the AEC method				
Pareto Solution #	First Objective Value (Cost)	Second Objective Value (TWU)		
1	7127	0		
2	6513	20		
3	6110	20		
4	5510	30		
5	5178	40		
6	4650	60		
7	4045	130		
8	3690	180		
9	3625	230		
10	3310	280		
11	3150	340		
12	3098	370		

Table 5



~~~ .1



Table 6

| Irade-Off between objectives using the NGSAII method |                              |                              |  |  |
|------------------------------------------------------|------------------------------|------------------------------|--|--|
| Pareto Solution #                                    | First Objective Value (Cost) | Second Objective Value (TWU) |  |  |
| 1                                                    | 7349                         | 0                            |  |  |
| 2                                                    | 6513                         | 20                           |  |  |
| 3                                                    | 5890                         | 30                           |  |  |
| 4                                                    | 5178                         | 40                           |  |  |
| 5                                                    | 4850                         | 50                           |  |  |
| 6                                                    | 4245                         | 140                          |  |  |
| 7                                                    | 3750                         | 190                          |  |  |
| 8                                                    | 3625                         | 230                          |  |  |
| 9                                                    | 3512                         | 280                          |  |  |
| 10                                                   | 3150                         | 340                          |  |  |
| 11                                                   | 3098                         | 370                          |  |  |



Fig. 3. Pareto front of the NGSAII method

To compare the results of the precise AEC method and the NSGAII metaheuristic method, the figure below displays the Pareto fronts of both methods. Although the problem size is small and can be effectively solved by the precise AEC method, it can be observed that the AEC method's Pareto front relatively dominates over the NSGAII method's Pareto front. However, the NSGAII method also shows relatively acceptable performance for this numerical example, with its Pareto front being close to a portion of the globally obtained Pareto front from the AEC method.

In practice, a solution needs to be selected from the set of Pareto solutions, and decisionmakers make this selection by balancing the obtained set of Pareto solutions [64-65]. The following figure depicts a space of Pareto solutions, and it is suggested to choose a selected solution from this space as the cost increase rate is significantly higher compared to the coverage increase rate. Figure 4 illustrates the optimal location and allocation of inspection centers to the demand regions based on the proposed efficient solution.



**Fig. 4.** Characteristics of the Proposed Pareto Solution Space for Selecting a Pareto Solution

In the following the RPP approach is evaluated in addressing the uncertainty in the problem. Two indicators, "deviation from optimality" (Figure 5) and "breach of restrictions" (Figure 6) are among the key measures used to assess the performance of optimization approaches under uncertain conditions. To utilize these indicators, the uncertain parameters are simulated 20 times, and the performance of the proposed RPP approach is evaluated. In the robust possibility approach (II), the value of  $\alpha$  is predetermined (here,  $\alpha$  is considered 95%). Ultimately, the value of  $\alpha$  is treated as a variable obtained from solving the model ( $\alpha$ =67%). Based on the results obtained in the following figures, it can be observed that the oscillation of optimality in the proposed robust approaches is much lower than the nominal value approach. Secondly, the proposed robust possibility approaches significantly reduce constraint violation compared to the nominal value approach, leading to a reduced risk in decision-making.



**Fig. 5.** The indicators of deviation from optimality for robust and nominal approaches



Fig. 6. The indicators of constraint violation for robust and nominal approaches

#### 4.3. Validation and Evaluation of the Proposed Solution

In the previous section, a case study was presented, and the solution results demonstrated that the obtained Pareto front from the proposed metaheuristic method closely approximates the exact front, indicating its convergence towards the global Pareto front. Therefore, the performance of the proposed metaheuristic solution for this numerical example was deemed acceptable. This section will discuss a more comprehensive validation of the proposed metaheuristic method.

In the proposed genetic algorithm for solving the problem, four factors/parameters, namely MaxIt (Number of iterations), POP (Initial population size), PC (Crossover rate), and PM (Mutation rate), must be adjusted at optimal levels. For this purpose, initially, three levels (low (1), medium (2), and high (3)) are defined separately for each parameter for solving the problem in small and large dimensions, as shown in Table 7 and Table 8. Then, a set of experimental tests for the Taguchi method is performed for the 4-factor case in 3 levels, resulting in 9 different scenarios, which are presented in the following tables (it should be noted that each experiment is repeated three times to reduce errors, and their averages are recorded).

| Defined levels for the parameters of the genetic algorithm for solving small-scale problems |         |            |          |  |
|---------------------------------------------------------------------------------------------|---------|------------|----------|--|
| GA Parameter                                                                                | Low (1) | Medium (2) | High (3) |  |
| MaxIt                                                                                       | 70      | 100        | 130      |  |
| POP                                                                                         | 30      | 40         | 50       |  |
| PC                                                                                          | 0.75    | 0.8        | 0.9      |  |
| PM                                                                                          | 0.15    | 0.20       | 0.25     |  |

Table 7

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Table 8

Table 9

| Defined levels for the parameters of the genetic algorithm for solving large-scale problems |         |            |          |  |
|---------------------------------------------------------------------------------------------|---------|------------|----------|--|
| GA Parameter                                                                                | Low (1) | Medium (2) | High (3) |  |
| MaxIt                                                                                       | 150     | 200        | 300      |  |
| POP                                                                                         | 70      | 100        | 150      |  |
| PC                                                                                          | 0.70    | 0.75       | 0.80     |  |
| PM                                                                                          | 0.20    | 0.25       | 0.30     |  |

Taguchi method is one of the techniques for parameter adjustment (optimal control of factors) introduced in 1978. The Taguchi method has two significant advantages. First, it does not require examining all possible experiments for the factors; only a specific fraction of experiments is investigated. Second, it extracts an appropriate amount of information from the investigated fraction, allowing for adjusting factors using relatively good information (Table 9).

| Experiments were designed using the Taguchi method for parameter tuning |       |     |    |    |  |
|-------------------------------------------------------------------------|-------|-----|----|----|--|
| Experiment Number                                                       | MaxIt | POP | PC | PM |  |
| 1                                                                       | 1     | 1   | 1  | 1  |  |
| 2                                                                       | 1     | 2   | 2  | 2  |  |
| 3                                                                       | 1     | 3   | 3  | 3  |  |
| 4                                                                       | 2     | 1   | 2  | 3  |  |
| 5                                                                       | 2     | 2   | 3  | 1  |  |
| 6                                                                       | 2     | 3   | 1  | 2  |  |
| 7                                                                       | 3     | 1   | 3  | 2  |  |
| 8                                                                       | 3     | 2   | 1  | 3  |  |
| 9                                                                       | 3     | 3   | 2  | 1  |  |

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In each Taguchi experiment, the best-achieved response for the problem is selected and recorded. The time taken to solve each experiment is also mentioned alongside it. Therefore, in response to each experiment a value of  $F_1^i$  represents for the first objective value,  $F_2^i$  for the second objective value of the problem. RT<sub>i</sub> represents the execution time of the algorithm. Since the response variable in each Taguchi experiment should be univariate, we consider the normalized combination of the main criterion/response,  $F_1^i$ , and  $F_2^i$ , which is calculated as follows:  $Q_i = \frac{1}{2} \left( \frac{F_1^i - m_1}{M_1 - m_1} + \frac{F_2^i - m_2}{M_2 - m_2} \right)$  Where  $M_1$  and  $m_1$  represent the minimum and maximum values of the first objective, respectively, and  $M_2$  and  $m_2$  represent the maximum and minimum levels of the second objective, respectively. It is clear that the defined criterion for experiment i is a number between 0 and 1, and the closer it is to 0, the better the response of that experiment. Results are shown in Table 10 and Table 11 for small-scale and large-scale problems respectively.

#### Table 10

| Obtained results from genetic algorithms for small-scale problems |      |     |        |           |  |  |
|-------------------------------------------------------------------|------|-----|--------|-----------|--|--|
| Experiment Number                                                 | Cost | TWU | Q      | Algorithm |  |  |
|                                                                   |      |     |        | Run Time  |  |  |
| 1                                                                 | 5200 | 210 | 0.9389 | 12.23     |  |  |
| 2                                                                 | 4905 | 185 | 0.5090 | 15.50     |  |  |
| 3                                                                 | 4537 | 177 | 0.2195 | 16.60     |  |  |
| 4                                                                 | 5310 | 207 | 0.9681 | 15.12     |  |  |
| 5                                                                 | 4414 | 163 | 0.0022 | 17.43     |  |  |
| 6                                                                 | 4421 | 170 | 0.0806 | 19.76     |  |  |
| 7                                                                 | 5018 | 191 | 0.6357 | 18.34     |  |  |
| 8                                                                 | 4621 | 180 | 0.2981 | 21.98     |  |  |
| 9                                                                 | 4410 | 164 | 0.0106 | 22.19     |  |  |

| Obtained results from | genetic algorithms for | small-scale problems |
|-----------------------|------------------------|----------------------|
|                       |                        |                      |

#### Table 11

Obtained results from genetic algorithms for large-scale problems

| Experiment Number | Cost  | TWU | Q      | Algorithm |
|-------------------|-------|-----|--------|-----------|
|                   |       |     |        | Run Time  |
| 1                 | 20151 | 903 | 1.0000 | 54.35     |
| 2                 | 18207 | 713 | 0.4766 | 83.78     |
| 3                 | 18001 | 720 | 0.4883 | 113.90    |
| 4                 | 19107 | 763 | 0.6250 | 72.56     |
| 5                 | 18303 | 731 | 0.5238 | 118.67    |
| 6                 | 17940 | 711 | 0.4644 | 174.23    |
| 7                 | 1841  | 746 | 0.1114 | 101.12    |
| 8                 | 17602 | 701 | 0.4304 | 189.45    |
| 9                 | 17801 | 705 | 0.4457 | 231.05    |

To determine the optimal level of each factor in both small-scale and large-scale problems, we first consider the signal-to-noise ratio (S/N) criterion (Figure 7 and Figure 9) in the smallerthe-better form. The level of each factor that shows a significantly higher S/N ratio compared to other levels is considered optimal for that factor. If there is no significant difference based on this criterion, the second criterion is considered, which is the execution time of the algorithm (Figure 8 and Figure 10).







**Fig. 8.** The execution time criterion is considered in the Taguchi method for parameter adjustment in small-scale problems









To adjust the parameters of the NSGAII method using the Taguchi approach in small-scale problems, the following parameter settings are determined based on the given information:

- MaxIt: The parameter MaxIt achieves the highest S/N ratio at level 2, which is set to 100.
- POP: Although there is no significant difference between levels 2 and 3 for the parameter POP, considering the execution time criterion, it is set to level 2, resulting in POP=40.
- PC and PM: PC and PM parameters have the highest S/N ratio at levels 3 and 1, respectively, so PC=0.9 and PM=0.15.

Similar parameter adjustments using the Taguchi method can be applied to solving large-scale problems, and the corresponding results are presented in Table 12.

#### Table 12

| Туре                 | PM   | PC   | POP | MaxIt |
|----------------------|------|------|-----|-------|
| Small-Scale Problems | 0.15 | 0.9  | 40  | 100   |
| Large-Scale Problems | 0.25 | 0.80 | 100 | 300   |

## 4.4 Solving experimental problems and evaluating solution methods

Each small-scale experimental problem has been successfully solved using the precise AEC method (Table 13). The AEC method yielded a specific number of accurate Pareto solutions. Subsequently, these experimental problems were resolved using the proposed metaheuristic method, resulting in identifying the Pareto front (Table 14). Evaluating the results based on various criteria, we observe that the CS criterion for the proposed metaheuristic algorithm is close to 0. This indicates that its solutions are competitive but slightly inferior to the exact solutions obtained by the AEC method.

#### Table 13

Experimental scenarios for the small-scale problems

| Experiment Number | Ν  | Μ  | К | S  |  |
|-------------------|----|----|---|----|--|
| 1                 | 10 | 12 | 3 | 5  |  |
| 2                 | 12 | 15 | 3 | 6  |  |
| 3                 | 15 | 15 | 3 | 7  |  |
| 4                 | 17 | 20 | 3 | 8  |  |
| 5                 | 19 | 25 | 3 | 10 |  |
| 6                 | 23 | 25 | 3 | 10 |  |
| 7                 | 25 | 30 | 3 | 12 |  |
| 8                 | 28 | 30 | 3 | 14 |  |
| 9                 | 32 | 35 | 3 | 15 |  |
| 10                | 35 | 35 | 3 | 20 |  |

| Experimental | scenarios for the large-scale problems |  |
|--------------|----------------------------------------|--|
| LAPCINICIU   | section to the large scale problems    |  |

| Experiment Number | Ν   | Μ   | К | S  |
|-------------------|-----|-----|---|----|
| 1                 | 40  | 40  | 5 | 20 |
| 2                 | 50  | 55  | 5 | 20 |
| 3                 | 60  | 70  | 5 | 30 |
| 4                 | 70  | 70  | 5 | 30 |
| 5                 | 80  | 90  | 5 | 40 |
| 6                 | 90  | 90  | 5 | 40 |
| 7                 | 100 | 120 | 5 | 50 |
| 8                 | 120 | 120 | 5 | 50 |
| 9                 | 140 | 150 | 5 | 60 |
| 10                | 150 | 150 | 5 | 60 |

Additionally, the MID criterion indicates the similar quality of solutions in the Pareto front for both methods. The NOS and NS\_CS criteria demonstrate that the proposed metaheuristic method provides acceptable diversity and quality of Pareto solutions for solving smalldimensional experimental problems. The RT criterion shows that this method efficiently solves each of the small-dimensional experimental problems within a reasonable time frame of approximately less than 20 minutes. On the other hand, none of the large-dimensional problems were solved within an acceptable time frame by the precise method. However, the proposed metaheuristic method successfully solved these problems in less than 3 hours and produced many acceptable Pareto solutions. Since both the precise and metaheuristic methods effectively solved the small-dimensional experimental problems, and the results indicate that the performance of the metaheuristic algorithm is comparable to the exact solution, the solutions obtained by the metaheuristic method can be considered of acceptable quality for large scales. Given that the AEC method is unsuitable for large scales, the proposed metaheuristic method is a suitable alternative (Table 15 and Table 16).

Table 15

Comparison of the proposed solution approaches based on evaluation criteria (small-scale problems)

| Experiment | NS_CS         | NOS      | NOS   | MID      | MID    | CS            |
|------------|---------------|----------|-------|----------|--------|---------------|
| Number     | (AEC, NSGAII) | (NSGAII) | (AEC) | (NSGAII) | (AEC)  | (AEC, NSGAII) |
| 1          | 4             | 4        | 4     | 150.83   | 150.83 | 0             |
| 2          | 4             | 4        | 4     | 134.60   | 134.60 | 0             |
| 3          | 4             | 6        | 5     | 203.12   | 221.89 | 0.33          |
| 4          | 8             | 8        | 7     | 287.24   | 304.05 | 0             |
| 5          | 8             | 10       | 11    | 275.31   | 287.51 | 0.20          |
| 6          | 13            | 13       | 13    | 390.64   | 351.43 | 0             |
| 7          | 12            | 14       | 16    | 430.65   | 400.85 | 0.14          |
| 8          | 14            | 15       | 17    | 494.65   | 531.15 | 0.07          |
| 9          | 15            | 15       | 17    | 559.08   | 560.13 | 0             |
| 10         | 17            | 17       | 19    | 604.15   | 587.42 | 0             |

#### Table 16

NSGA-II metaheuristic method performance for large-scale problems

| Experiment Number | Run Time (Min) | NOS (NSGAII) | MID (NSGAII) |  |  |
|-------------------|----------------|--------------|--------------|--|--|
| 1                 | 20.56          | 25           | 820.65       |  |  |
| 2                 | 26.87          | 31           | 928.76       |  |  |
| 3                 | 33.9           | 28           | 1070.67      |  |  |
| 4                 | 41.41          | 30           | 1324.59      |  |  |
| 5                 | 53.43          | 35           | 1351.73      |  |  |
| 6                 | 70.12          | 37           | 1377.90      |  |  |
| 7                 | 90.31          | 40           | 1630.59      |  |  |
| 8                 | 120.86         | 42           | 1635.80      |  |  |
| 9                 | 150.43         | 38           | 1746.24      |  |  |
| 10                | 196.98         | 41           | 1875.79      |  |  |

## 4.5. Conclusion

To demonstrate the hypothesis, a numerical case study was conducted on the location of technical inspection centers in a specific region of Tehran. Initially, a small-scale problem instance was defined, and the solutions obtained from the proposed approaches were evaluated and analyzed. The results indicated the satisfactory performance of the accurate AEC method and the proposed metaheuristic approach. Additionally, the proposed RPP method showed promising results in handling uncertainty.

Following the successful resolution of the case study, various problem variations were presented in different dimensions. By solving these experimental problems and comparing the performance of the accurate AEC method with the metaheuristic NSGA-II method using evaluation criteria such as MID, NOS, and NS\_CS, it was observed that NSGA-II performed acceptably in small dimensions when compared to the accurate AEC method. Moreover, in larger dimensions where the AEC method is not applicable, NSGA-II proved to be a viable alternative. The results obtained from solving the large-dimensional problems demonstrated the capability of the proposed metaheuristic algorithm to solve the problem and consistently obtain a significant number of Pareto solutions.

## 5. Results

Location allocation is an efficient and essential management aspect, encompassing one of the most consequential decision-making processes. By giving it due attention, businesses can reduce costs and enhance the success of their industrial units, which are subject to numerous influencing factors. The primary goal of location allocation is to identify a range of suitable spatial options for a specific purpose. Finding the right location for a venture within a particular geographic area is a crucial step in large-scale implementation projects. Location allocation decisions primarily fall into the long-term and strategic category, necessitating consideration of various conflicting objectives. In addition to customers, facilities, space, and distance, other factors significantly impact location-allocation models.

This research focuses on evaluating and applying the RPP approach for handling uncertainty in problem control. Two crucial criteria, namely "deviation from optimality" and "constraint violation," are utilized to assess the performance of optimization methods under uncertain conditions. To evaluate these criteria, uncertain parameters are simulated 20 times, and the effectiveness of the proposed RPP approach is analyzed. The results indicate that the proposed robust approaches exhibit significantly lower optimality oscillation than the nominal value approach. Moreover, these robust approaches effectively reduce constraint violations, reducing decision-making risks.

Furthermore, a comparative analysis is conducted between the precise AEC and the NSGAII metaheuristic methods using various test problems in different dimensions. The evaluation is based on MID, NOS, and NS\_CS criteria. The findings reveal that NSGAII performs acceptably well in small dimensions compared to the precise AEC method. In large dimensions where the AEC method fails to solve the problem, NSGAII can be employed as an alternative. The results of solving large-scale problems demonstrate the capability of the proposed metaheuristic algorithm, which consistently yields numerous Pareto solutions.

Finally, as the most critical managerial recommendations derived from the numerical results, the following points can be highlighted:

- Although there is a direct correlation between the costs of establishing technical inspection centers and the covered areas, the cost increase does not significantly affect the extent of coverage for the regions requiring technical inspection. This is the constraint on the maximum number of constructions of technical inspection centers.
- A robust probabilistic planning approach reduces decision-making risks and significantly decreases the selected locations for establishing technical inspection centers. The two robustness criteria, robust optimality, and robustness achievability, confirm this observation.
- Optimal locations for deploying technical inspection centers are not necessarily areas with high potential populations. The demand rate for technical inspection also plays a crucial role.
- Considering a specific coverage radius ensures that the regions requiring coverage have adequate access to technical inspection centers within an acceptable timeframe.

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## **Conflicts of Interest**

The authors declare no conflicts of interest.

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